Models of DM

- The taxonomy of DM models is rich. How many I think?
  - neutralino (bino, wino, higgsino)
  - LTOP
  - singlino
  - LPop
  - sneutrino
  - excited DM (XDM)
  - axion
  - axino
  - kK photon
  - inert doublet
  - scalar singlet

- Broad division into two classes (abundance mech.)

<table>
<thead>
<tr>
<th>Thermal</th>
<th>Non-Thermal</th>
</tr>
</thead>
<tbody>
<tr>
<td>WIMP's (usually)</td>
<td>decay products</td>
</tr>
<tr>
<td>gets most attention</td>
<td>coherent scalar fields</td>
</tr>
<tr>
<td></td>
<td>very-very weakly coupled</td>
</tr>
<tr>
<td></td>
<td>examples later</td>
</tr>
</tbody>
</table>

- There are hybrids: super-WIMPs (decay daughter of thermal parents) (platypus?)

- A sub-class of Non-Thermal Models grows into a class of its own:
  - Asymmetric DM
  - $\Omega_{dm} \sim 5 \Omega_{b}$
Coherent Scalar Fields & Axions

Consider a light scalar field:

Eq. of motion:

\[ \ddot{\phi} + 3H \dot{\phi} + m^2 \phi = 0 \]

- Famously, when \( H \gg m \) the field is in "slow-roll" (a.k.a. stuck)
- When \( m \) is small the field rolls and oscillates with a frequency \( m \)
- These oscillations store energy (but no momentum if \( \dot{\phi} = 0 \)). How does it evolve?

\[ \mathcal{L}_\phi = \frac{\dot{\phi}^2}{2} - \frac{m^2 \phi^2}{2} \]

and \( \mathcal{L}_\phi = \ddot{\phi} \phi + m^2 \phi \phi \)

- Rewrite e.o.m.:

\[ \dddot{\phi} + m^2 \dot{\phi} + 3H \ddot{\phi} = 0 \]

\[ \bar{L}_\phi = \frac{\mathcal{L}_\phi}{\int \mathcal{L}_\phi} \]

\[ \bar{L}_\phi = \frac{\dot{\phi}^2}{2} - \frac{m^2 \phi^2}{2} \]

\[ \int \bar{L}_\phi = \int \left( \frac{\dot{\phi}^2}{2} - \frac{m^2 \phi^2}{2} \right) = 0 \]

\[ \frac{\dot{a}^2}{a} + 3 \dot{a} a = 0 \]

\[ \frac{\dot{a}^2}{a} + 3 \dot{a} a = 0 \]
Coherent scalar (continued):

So, \( \int a^3 = \text{constant} \)

\[ \downarrow \]

the energy stored in this field
redshifts like dark matter!!

- this is a pressureless fluid \( \Rightarrow \) clumps!

\( \left( p_a \right) = \frac{1}{2} m \Phi^2 \Rightarrow 0 \)

What determines \( \Lambda_{	ext{obs}} \):

1. the initial value of \( \Phi, \Phi_i \)

2. the redshift at which oscillations
   begin, \( H(T_i) \sim M_\Phi \).

OK, let's try it out:

- take a scalar of mass \( M_\Phi \) and start
  with a "generic" vev, \( \langle \Phi \rangle \sim M_{\Phi} \).

(* \( H(T_i) \sim M_\Phi \))

\[ S_\Phi(T_i) \sim M_\Phi^2 \langle \Phi \rangle^2 = M_\Phi^2 M_{\Phi}^2 \]
\[ \Rightarrow \text{energy density from } \Phi \text{ dominates at } T_i \]

\[ \text{? = hey! what about growth of structure} \]
\[ \text{during radiation domination?} \]

"\( \Lambda_{	ext{obs}} \) problem" \( \Rightarrow \) Bad on candidates
Coherent Scalar & Axions:

- If DM is a coherent scalar, it must:
  - have a “non-generic” vev \((\Phi \ll M_{pl})\).
  - or, get its mass “late” (when \(\phi = m\) already).

- the Axion (satisfies both, usually) \((\Phi \ll M_{pl})\)

- Motivated by the Strong CP problem.

- The axion is a PNB. Its a pseudo due to QCD instanton effect. Give the axion a mass:

  \[ m_{a} \sim \frac{\Lambda_{QCD}}{f} \]

- Axion mass “turns on” at QCD phase transition, \(T \sim \Lambda_{QCD} \)

- A “generic” axion vev is of order \(f \sim M_{pl} \)

  A careful calculation (called Dine-Holl\’s ch 10):

  \[ 2\pi \alpha = \left( \frac{f}{10^{-15} \text{ GeV}} \right)^{1.175} \]  

  - The 1.175 comes from non trivial mass turn-on,

  - \(\theta_{i}\) initial value of \(\frac{\alpha}{f}\), QCD angle.

  e.g. from M\(_{\text{SB}}\) \(\theta_{i} \sim 10^{-2}\) works well.
Thermal Dark Matter:

- Add a new particle, $\chi$, to the primordial soup.
- Include interactions:
  - $\chi \chi \leftrightarrow f \bar{f}$ (if $f$ is any SM particle)
- Annihilation goes back and forth:
  - In equilibrium, for $T \gg m_{\chi}$, $n_{\chi} \sim T^3$
  - for $T \approx m_{\chi}$, $n_{\chi} \sim (m_{\chi} T)^{3/2} e^{-m_{\chi}/T}$
  - $\leftarrow$ steps, but $\rightarrow$ continues
- Define $Y_\chi = \frac{n_{\chi} \Omega}{\rho_{\text{c}}}$, $\chi = \frac{m_{\chi}}{T}$
  - coupling density a time variable

\[ Y_\chi \]
Freeze-out:

- At some point $x$'s will be too rare to find each other: freeze-out.

- After freeze-out $n_x$ dilutes with expansion

$$n_x \propto a^3 \Rightarrow y_x = \text{const}$$

This curve is described by a Boltzmann equation. Let's not solve one here... get intuition instead:

- Compute $N_{\text{ann}} = \langle \text{average # of annihilations} \rangle$ of $\chi$ particles undergone beyond time $t_i$ if $N_{\text{ann}} \ll 1 \Rightarrow$ freeze-out.

**Rate:**

$$\Gamma \sim n_x (\sigma \nu)$$  \hspace{1cm} (Born, cross section)

$$v = \text{relative velocity}$$

$$N_{\text{ann}} = \int_{t_i}^{t} \Gamma dt = \int_{t_i}^{t} \frac{\Gamma dt}{\frac{d}{dt} \log T} = \int_{\frac{t_i}{H}}^{\frac{t}{H}} \frac{d(\log T)}{H}$$

$$N_{\text{ann}} \sim \frac{t - t_i}{H}$$

(in an e-fold of expansion)

$$N_{\text{ann}} \sim \langle \frac{\Gamma}{H} \rangle$$
Freeze-out (continued):

- Use \( H(t) = H(t_0) \left( \frac{t}{t_0} \right)^n \)

\[
\Gamma = \eta_0 \nu_0 \alpha T^n \quad \text{(e.g., } \eta_0 \nu_0 \alpha \propto T^2) \]

\[
N_m = \int_{t_0}^{\infty} \frac{\Gamma(t)}{H(t)} \frac{T^{m+1}}{T_0^{m+2}} dT = \frac{\Gamma(t)}{H(t)} \frac{1}{n-2}
\]

- once \( \Gamma < H \)
- \( N_m \) is small.

Freeze-out happens when \( \Gamma \approx H \).

- calculate \( T_0 \) (in \( x_f \))

\[
\eta_x (\sigma v) \sim \frac{T_{t_0}^2}{M_{Pl} \eta}
\]

\[
\left( \frac{m^2}{x_f} \right)^2 e^{x_f} \langle \sigma v \rangle \sim \frac{m_x^2}{M_{Pl} x_f^2}
\]

\[
\log \left[ \frac{m^2}{x_f^2} \langle \sigma v \rangle \right] = x_f \sim \log \frac{m_x}{M_{Pl} x_f^2}
\]

\[
x_f \sim \log \left[ m_x \eta^2 \langle \sigma v \rangle \right] \sim 25 + \log \text{sensitivity} \]

- \( x_f \) is insensitive to details, \( \sim 25 \).

- DM is cold at freeze-out \( \check{\text{V}} \).
Freeze out (continued):

* the quantity we want is a remaining energy

$$\Omega_{\text{DM}} \propto m^2 \frac{m_x}{\langle \sigma v \rangle} \sim \frac{H(t)}{\langle \sigma v \rangle} \frac{1}{M_{\text{Pl}}} \frac{1}{f} \langle \sigma v \rangle$$

$$\Omega_{\text{DM}} \propto \frac{x_f}{M_{\text{Pl}} \langle \sigma v \rangle}$$

* Relic density is insensitive to mass.
  only sensitive to $\langle \sigma v \rangle$!

* After number plugging

$$\Omega_{\text{DM}} h^2 = 0.1 \left( \frac{x_f}{0.1} \right) \left( \frac{g_*}{80} \right)^{-\frac{1}{4}} \left( \frac{\langle \sigma v \rangle}{3 \times 10^{-26} \text{cm}^3/\text{s}} \right)^{-\frac{1}{4}}$$

* in natural units $\langle \sigma v \rangle \sim$ EW cross section

  $\sim 0.1 \text{ pb}$

  or $\frac{\alpha^2}{1000 \text{ GeV}^2}$

* We have plenty of other reasons to expect
  new physics to come in at that scale!!

(this is known as the "WIMP miracle"
I hate this name.)