Composite Higgs - Lecture 4

Introduction

Focus: What models (naturally) include a light composite scalar? \(125 \text{ GeV}\)

How would we know?

Toolbox: (1) Effective Field Theory
   Naive Dimensional Analysis
(2) Large-\(N\)/Quasi Perturbative Method
(3) Chiral Lagrangians
   [ AdS/CFT, Nonperturbative SUSY, etc... ]

What is compositeness?

\[ \begin{array}{c}
\text{QCD} \\
\text{Natural Scale} \\
\text{Fundamental Hierarchy}
\end{array} \]

\[ \begin{array}{c}
E \\
M_\Delta \\
M_{\text{Planck}} \\
M_p \\
M_T
\end{array} \]

\[ M_T \ll M_p, \mu_p, M_\Delta \text{ (16 eV)} \]

Form Racetrack, "size", etc \(O(16 \text{ eV})\).
Goal! Construct theory in which the "Higgs" is composite with \( M_x \ll \Lambda \).

* How do we describe such a theory, \( @ E \ll \Lambda \)?

** Effective Field Theory

* Why would \( M_x \ll \Lambda \)?

\[ \text{Symmetry} \Rightarrow \text{What are the symmetries of the "Higgs"?} \]

or "Tuning" \( \Rightarrow \) A conspiracy of high-energy points.

** Effective Field Theory

QFT Reconciles QM w/ Relativity:

\[ \begin{cases} \text{Any local, Lorentz-invariant, Hermitian} \\ \text{QFT with a finite # of fields} \Rightarrow \\ \text{Unitary, CPT-invariant S-matrix,} \\ \text{Cluster Decomposition} \end{cases} \]

Following Landau, we assume that we can take experimental data (a unitary S-matrix) and construct a QFT description.
There can be many theoretical descriptions which describe the same S-matrix:

1. They will share the same global symmetries, conserved quantities.

2. They need not have the same redundant variables → Gauße symmetries → different "degrees of freedom".

3. Coupling constants are not physical quantities → obvious if different descriptions, even in some descriptor couplings are regularization/renormalization dependent.

"Fundamental" vs. "Composite" not as important as "Strong" vs "Weak" can depend on energy range of interest.
Example: Complex scalar field, UV cutoff

\[ \frac{\partial^2 \phi}{\partial x^2} - \frac{1}{\epsilon^2} \left( \frac{d\phi}{dt} \right)^2 = 0 \]

\[ \phi \rightarrow e^{i\phi} \]

All four amplitudes (unit, P, CP, T)

Chiral symmetry

\[ P, \text{anomaly} \]$$

\[ T \text{ problem, strong CP} \]

4-0 soluble gauge Theory

\[ \gamma \]"Spread - Copy" = Not so much

"Spread - Copy" = Not so much

"Spread - Copy" = Not so much

"Spread - Copy" = Not so much

Effective chiral Lagrangian: $$\chi \rightarrow P, \bar{P}$$

Hidden chiral symmetry

Not asymptotic

Not asymptotic

Asymptotic

Asymptotic

Asymptotic

Asymptotic

Asymptotic

Asymptotic

Asymptotic
Rules: (1) That which is not forbidden by symmetry, is required. Different, possibly, with SUSY due to non-renormalizable terms. Depends on your view of the "high-energy" completion.

(1) No small number \( \Rightarrow \) What is small?

(2) Use \( \Lambda \) to relate couplings for dimensionless

\[ \lambda'(\Lambda) = \lambda(\Lambda') \]

\[ \kappa'(\Lambda) = \kappa(\Lambda') \]

Wilson: Suppose we are interested in \( \mu \ll \Lambda \Rightarrow \) "Integrate out" states with \( \Lambda' < \Lambda < \Lambda \).

\[ \Lambda \Rightarrow \Lambda' \]

\[ m^2(\Lambda) \Rightarrow m^2(\Lambda') \]

\[ \lambda(\Lambda) \Rightarrow \lambda(\Lambda') \]

\[ \kappa(\Lambda) \Rightarrow \kappa(\Lambda') \]

In practice, this is done order by order in couplings, and in dimension of operator \( \langle \Phi^2 / \Lambda^2 \rangle \).

Exception! Lattice gauge theory.

\( \Rightarrow \) Behavior of they encode in RG flow of parameters, scale to \( \overline{\Lambda} \).
Perturbative analysis $\Rightarrow$

\[ K(\lambda') = \frac{\lambda'^2}{\lambda^2} K(\lambda) + \cdots \to 0 \quad \text{Irrelevant coupling} \]

\[ \frac{1}{\Lambda(\lambda')} = \frac{1}{\Lambda(\lambda)} + b \ln \left( \frac{\lambda'}{\lambda} \right) \to \text{Marginal coupling} \]

\[ \Lambda \to m_{\text{phys}} \quad [\text{mass scale}] \]

\[ \Lambda = M_{\text{GUT}} \]

\[ m^2(\lambda') = m^2(\lambda) + c (\lambda^2 - \lambda'^2) + \cdots \]

\[ \to \infty \quad \text{Naturalness/Hierarchy Problem} \]

Need to "Tune" high energy physics so that $m_H^2 (m_H^2)$ stays low$^\text{a} \leq 1 \text{TeV}^2$

Renormalizability $\equiv$ Finite # of Marginal/Relent Coupled

$\equiv$ Renormalizable by "Fon-Count" Analysis can be done non-perturbatively as well $\equiv$ Lattice Gauge Theory

\[ \text{Need } m_H^2 \ll \frac{1}{\alpha}, \quad \text{"Continuum limit", 2nd order P.T.} \]
Our Viewpoint Here:

(1) All descriptions of nature are "Effective Field Theories," valid to some finite observed precision.

(2) Constructed order by order in couplings (usually)

(3) Implicitly depend on some scale $\Lambda_{UV}$, at which the description of the theory may change, 
# of "relevant" couplings explodes,

⇒ Dual expansion in couplings and (implicitly) in $(\mu^2/\Lambda_{UV}^2)$.

More

$QCD$: $X$ Lagrangian, $\Lambda_{UV} = 16\text{eV}$

SM one Higgs Doublet: $\Lambda_{UV} = ?$

"Fundamental" Higgs: $\Lambda_{UV} \to \infty$

Triviality forbids this.

But, if $\Lambda_{UV} = M_{WZ}$ or $\sim 100$ TeV? 

May not make sense.
Weinberg/Witten: A massless spin-1 particle couples only if it couples like a gauge boson, and has a conserved gauge-invariant.

[Similar true for gauge spin-2/graviton]

Consequence: Observe spin-1 gauge bosons (W, Z), have

\[ \frac{M_W, Z}{\langle \phi \rangle} \Rightarrow \text{limit} \frac{1}{\langle \phi \rangle} \]

\[ \frac{\text{SU}(2) \times \text{U}(1)}{\text{gauge theory}} \]

\[ \text{SSB} \]

\[ \Rightarrow \text{Explains universality of gauge-couplings} \]

Universal + Customary Symmetry

\[ \Rightarrow \text{Most of the successes of SM} \]

(??)

SM Higgs as an Effect. Theory

\[ \mathcal{L} = D_{\mu} \phi^+ D_{\mu} \phi - \frac{1}{2} (\phi^2 - \frac{v^2}{2})^2 + \text{fermion terms} + \text{gauge} \]

\[ \phi = \phi^+ + i \phi^0 \]

Problems: \[ \phi^0 = 162 \text{ GeV} \text{ Translates to } Z - 126 \]
Define \( \Phi = (\varphi^+ \varphi) \).

Show

\[ D^\mu \varphi^+ D_\mu \varphi = \frac{1}{2} \text{Tr} \, D^\mu \Phi^+ D_\mu \Phi \]

\[ D^\mu \bar{\Phi} = D^\mu \varphi^+ + ig \bar{\psi} \gamma^\mu \varphi^+ - ig \varphi \gamma^\mu \bar{\psi} \]

\[ \psi^\mu \equiv \frac{\omega \gamma^9}{2}, \quad \varphi^\mu = \frac{B^\mu}{2}. \]

(c) \[ \psi^\mu \equiv \frac{\omega \gamma^9}{2}, \quad \Phi^+ \Phi = \left( \psi^\mu \psi \right) \left( \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right) \Rightarrow \Phi^+ \Phi = \rho \]

\[ \Rightarrow \left( \varphi^+ \varphi - \frac{v^2}{2} \right) = \left( \frac{\text{Tr} \Phi^+ \Phi}{2} - \frac{v^2}{2} \right) \]

\[ = \frac{1}{2} \left[ \text{Tr} \left( \Phi^+ \Phi - \frac{v^2}{2} \right) \right] \]

\[ \left( \varphi^+ \varphi - \frac{v^2}{2} \right)^2 = \frac{1}{2} \text{Tr} \left( \Phi^+ \Phi - \frac{v^2}{2} \Phi \right)^2 \]

\[ \langle \Phi \rangle = \frac{v}{\sqrt{2}} \left( \begin{array}{c} 1 \\ 1 \end{array} \right) = \frac{v}{\sqrt{2}} \Phi. \]
\[ \Rightarrow \text{(d) } V(\Phi) \text{ has an} \]
\[ \mathbf{S(U(2))_L \times S(U(2))_R} \text{ Global Symmetry} \]
\[ \Phi \rightarrow \zeta \in \mathbb{R}^4 \text{ Accidental Symmetry} \]
\[ \mathbf{S(U(2)_L = \mathbb{R} - yd \mathbf{S(U(2))_R}} \]
\[ \text{Gauged } U(1)_y \rightarrow T_3 \text{ Part of } S(U(2)_R} \]

**Parameterize**
\[ \Phi = \frac{v}{\sqrt{2}} e^{i(x)/\sqrt{\Sigma}} \]

\[ \Rightarrow \delta_{\Phi} = \frac{v^2}{2} \partial_u e^{i(x)/\sqrt{\Sigma}} e^{i(x)} \]
\[ + \frac{v^2}{4} e^{2i(x)} Tr D_u\Sigma + D_u\Sigma \]
\[ - \frac{\lambda v^4}{4} \left( e^{-1} \right)^2 + 4N \text{ Yukawa Term} \]

**Approximate Symmetries:**
\[ g, g' \rightarrow 0 \]

\[ \mathbf{S(U(2)_L \times S(U(2)_R) \rightarrow S(U(2)_R \leq S(U(2))} \]

**Scale Symmetry:**
\[ \delta(x) \rightarrow \delta(x) + v \ln A \]
\[ \gamma \rightarrow \sqrt{2} \gamma(x), A^a \rightarrow \lambda^a(x) \]
Problem: Write down the rest of the terms in $\mathcal{L}_\omega$. What are their properties only:

1. $SU(2)_r \times SU(2)_l$?
2. Scale Symmetry?
   - Which terms break this at tree-level?

Problem:

- Dimension $= 6$

1. What higher-order, high-dimension terms involve only $\phi$, or $\bar{\phi}$, can you write down which one $SU(2)_r \times SU(2)_l$ invariant, but break $SU(2)_r \times SU(2)_l$?

2. $SU(2)_r \times U(1)$ breaks the $SU(2)_r \times SU(2)_l$ symmetries. Introduce "spinors" which encode this:

   $Q^a \rightarrow L Q^a L^T$
   $Q^\dagger \rightarrow R Q^\dagger R^T$

3. What other independent $d=6$ terms are there? What are the effects?