Flavor Physics

What / Why / How

- Flavor physics: study the different types of quarks, “flavors”, their spectrum and transitions among them (interactions).
  
  More generally: leptons.
  
  Transitions: strengths, symmetries (e.g. CP/PT, ...).?

- Why? Richness (much to do and understand)
  
  - Stringent test of theory
  
  - Closely tied to all observed CP-violation?

- Methods involved are many/diverse: main challenge is strong interaction (invariant flavor physics)
  
  EFTs: electro-weak
  
  \( \chi-L_{\nu} \)
  
  HQET
  
  \( \text{SCE} \)...

  Symmetries

  Non-perturbative (\( \chi-P\nu\text{N} \))

Since SM works we will adopt it as our standard (loop) paradigm

\[
\mathcal{L} = \mathcal{L}_{\text{kin}} + \bar{\psi} \gamma^\mu D_\mu \psi - \left[ \lambda_{ij} H \bar{q}_i q_j + \lambda_{ij} H \bar{d}_i q_j + \lambda_{ij} H \bar{\ell}_i l_j + \text{h.c.} \right]
\]
\[ D_a = \partial_a + ig A^a \gamma^a + ig W^a \gamma^5 + ig \beta Y \]

**Flavor “symmetry”**

For \( X_{ij}^a = 0 \) we have \( U(3)^3 \) symmetry

\( \text{a U(1) is anomalous, we only need \( SU(3) \) factor} \).

\[ \begin{align*}
q_i \rightarrow U_{ij} \bar{q}_j, & \\
\bar{q}_i \rightarrow U_{ij} \bar{q}_j, \quad etc.
\end{align*} \]

Symmetry is broken explicitly by Yukawa interactions.

Can keep track of pattern of symmetry breaking by treating Yukawa couplings

**“spurions” (constant fields):**

\[ X_{ij} \bar{q}_i \rightarrow U^T \bar{\delta} x_{ij} \bar{\delta} \]

so, we take \( X_{ij} \rightarrow \bar{\delta} x_{ij} \)

and thus the term is invariant.

Better notation: \( \bar{\delta} x \)

\[ \begin{align*}
q_i \rightarrow U_{ij} \bar{q}_j, & \\
\bar{q}_i \rightarrow U_{ij} \bar{q}_j, \quad X \rightarrow U^T \bar{x} U
\end{align*} \]

\[ \begin{align*}
\bar{x} \bar{q}_i \rightarrow U_{ij} \bar{q}_j \left( U_{ij} \bar{x} U_{ij} \right) U_{ij} \bar{q}_j = \bar{x} \bar{q}_i
\end{align*} \]

and so on.

As we will see, new interactions that break this “symmetry” tend to produce rules of flavor transformations inconsistent with observation (absent tuning or large parametric suppression).

(Hence the usefulness of this symmetry).

We will be mostly concerned with hadronic (quark) physics (i.e. \( U(3)^3 \)).
The KM model of CP-violation and the CKM matrix (Review?)

In unitary gauge, $H = \frac{1}{\sqrt{2}} (\nu + \nu' )$. This gives mass terms for quarks and leptons:

$$
\mathcal{L} = \frac{1}{2} v \bar{\nu} \nu + \frac{1}{2} v \bar{d} d + \frac{1}{2} v \bar{u} u + \frac{1}{2} v \bar{d} d^\dagger d + \frac{1}{2} v \bar{u} u^\dagger u + \frac{1}{2} v \bar{d} d^\dagger d^\dagger,
$$

Diagonalize mass matrices (for simpler computation & interpretation):

This can be done by field redefinitions:

- Linear

- Leave $\nu$ invariant (properly normalized/diagonal kinetic terms)

  $\Rightarrow$ unitary transformations (not a $\mathbb{CP}^3$ transformation)

$$
V_L \rightarrow V_L' \nu_L, \quad V_u \rightarrow V_{u}' \nu_u, \quad V_d \rightarrow V_{d}' \nu_d
$$

Choose $V_L \equiv V_L^\dagger V_L = 1'$ diagonal, $V_u \equiv V_u^\dagger V_u = \lambda'$ diagonal, real, positive

Exercise: show this can always be done.

The $\mathcal{L} = \bar{\nu} \left( \begin{array}{c} i \gamma^5 \psi \end{array} \right) + \bar{d} \left( \begin{array}{c} i \gamma^5 \psi \end{array} \right) \nu = \mathcal{L}_m + \mathcal{L}_k$.

No $\bar{L}_L i \gamma^5 \nu_L \rightarrow \bar{L}_L i \gamma^5 \nu_L$.

What about gauge invariance?

$$
- \bar{\nu}_L \left( g_3 \gamma^{\mu} T^a + \frac{i g_5}{2} \gamma^{\mu} \right) \nu_L \rightarrow - \bar{\nu}_L V_{\nu}^\dagger (\text{diag}) V_{\nu} \nu_L = - \bar{\nu}_L (\text{diag}) \nu_L \quad \text{unchanged}
$$

Likewise for $\mathcal{L}_k$.

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$$
\mathcal{L}_k = \left( g_3 \phi^{\mu} T^a + \frac{i g_5}{2} \phi^{\mu} \right) \phi_L \rightarrow \bar{\phi}_L V_{\phi}^\dagger (\text{diag}) V_{\phi} \phi_L = - \bar{\phi}_L (\text{diag}) \phi_L
$$

But $\phi^0$ baryon is off-diagonal.

$$
\phi_L \left( g_3 \phi^{\mu} T^a + \frac{i g_5}{2} \phi^{\mu} \right) \phi_L \rightarrow \bar{\phi}_L V_{\phi}^\dagger (\text{diag}) V_{\phi} \phi_L = - \bar{\phi}_L (\text{diag}) \phi_L
$$

so $\phi^0 \phi^0 = \phi^0 \phi^0 = \phi^0 \phi^0 = \phi^0 \phi^0$.

And $\sigma^\tau = \frac{1}{2} \left( \begin{array}{c} \sigma^\tau \end{array} \right)$.

$$
\mathcal{L}_m = \frac{1}{2} \left( \begin{array}{cc} 0 & \phi^\dagger \phi \\
\phi^\dagger \phi & 0 \end{array} \right) q_L = \bar{q}_L \frac{1}{2} \left( \begin{array}{c} \phi^\dagger \phi \\
\phi^\dagger \phi \end{array} \right) \phi^\dagger \phi
$$

$CKM$ matrix $V = V_{\nu} V_{\phi}$. Feature (leaving $m_{\nu}$, diag, positive) $V \rightarrow \text{diag} \left( e^{\text{i} \phi_1}, e^{\text{i} \phi_2}, e^{\text{i} \phi_3} \right) V$

$V^\dagger V = 1$. If conditions $\approx 18.9 \pm \text{4 permutations}, -5 \pm \text{3 phases}$

$3 \approx 18.9 \pm \text{4 permutations}, -5 \pm \text{3 phases}$
\[ V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \]

1. One invariance principle: CP is violated in $\bar{u}_L V^{\dagger} W^+ d_L + \bar{d}_L V^+ W^- u_L$ under CP $\bar{u}_L \gamma^5 d_L \rightarrow \bar{d}_L \gamma^5 u_L$ and $W^+ \rightarrow W^- \bar{w}$; so $CP \Rightarrow V^T = V$.

Exercise: In QED $C: e^+ e^- \rightarrow -e^- e^+$ and $\nu \rightarrow -\bar{\nu}$, so $\bar{C} e^+ e^- \rightarrow \bar{C} e^- e^+$

In QCD? $C: \bar{q} T^a q \rightarrow \bar{q}(-T^a)^T q$, but $(-T^a)^T = -T^a$

What does $C$ symmetry mean in QCD? How does $T^a$ transform?

Exercise: If in $m_\nu$ (or $m_D$) two entries are equal, show $V$ can be brought into a real matrix (up, in $SU(3)$).

2. Precise knowledge of the elements of $V$ is necessary to constrain new physics (or to test the validity of the SM/cosmology). Will describe below how well we know it and why. But for now a sketch.

\[ V \sim \begin{pmatrix} e^0 & e^1 & e^2 \\ e^1 & e^2 & e^0 \\ e^2 & e^0 & e^1 \end{pmatrix} \quad \text{with} \quad e \sim 10^{-1} \]

3. $V^T V = V V^T = I \Rightarrow$ rows & columns of $V$ are orthonormal vectors.

\[ \sum V_{ui} V_{kj} = 0 \quad \text{for} \quad i \neq k : \text{sum of 3 complex} = 0 \quad \text{is a triangle in} \quad 2 \text{-plane} \]

<table>
<thead>
<tr>
<th>$\lambda_k$</th>
<th>$\sum V_{ij} V_{kj} = 0$</th>
<th>$\sim e^2$</th>
<th>Shape (normalized, s box = 1).</th>
</tr>
</thead>
<tbody>
<tr>
<td>12</td>
<td>$V_{ud} V_{cd} + V_{us} V_{cs} + V_{ub} V_{cb} = 0$</td>
<td>$e^0 + e^0 e^2 = 0$</td>
<td>$e^4$</td>
</tr>
<tr>
<td>13</td>
<td>$V_{ud} V_{cd} + V_{us} V_{cs} + V_{ub} V_{cb} = 0$</td>
<td>$e^0 + e^0 e^2 = 0$</td>
<td>$e^4$</td>
</tr>
<tr>
<td>1</td>
<td>$V_{ud} V_{cd} + V_{us} V_{cs} + V_{ub} V_{cb} = 0$</td>
<td>$e^0 + e^0 e^2 = 0$</td>
<td>$e^4$</td>
</tr>
</tbody>
</table>

These are "unitarity triangles." The most commonly discussed is in the 13 columns.
\[ V_{ud} V_{ub}^* + V_{cd} V_{cb}^* + V_{td} V_{tb}^* = 0 \quad \Rightarrow \quad \text{Diagram} \]

Dividing by middle terms:

\[ \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} + 1 + \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} = 0 \]

Other conventions:

\[ \beta = \delta_1, \quad \alpha = \delta_2, \quad \gamma = \delta_3 \]

Exercise: (i) Show that

\[ \beta = \arg \left( - \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right), \quad \alpha = \arg \left( - \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right), \quad \gamma = \arg \left( - \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right) \]

(iii) These are invariant under phase transformations of quarks.

(ie, under remaining arbitrariness) → physical

(iv) Area:

\[ \text{Area} = \frac{1}{2} \text{Im} \left( \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right) - \frac{1}{2} \text{Im} \left( \frac{V_{td} V_{tb}^*}{V_{cd} V_{cb}^*} \right) \]

\[ \text{Im} \left( \frac{V_{ud} V_{ub}^*}{V_{cd} V_{cb}^*} \right) \quad \text{(under phase transform)} \]

\[ J = \text{Jadach-Jerzy invariant} \]

Note that

\[ \text{Im} V_{ij} V_{kj}^* V_{ij}^* V_{kj} = J \left( \delta_{ij} \delta_{kj} - \delta_{ik} \delta_{kj} \right) \]

is the commutator of all triangles.

The normalized triangle, have area \( J/|\text{Im} V_{ij}| \).

As we'll see, the area of the normalized triangle dictates the \( \beta \)-asymmetries.

(smaller, order \( e^{-} e^{+} \) for squashed triangles, order 1 for fat triangles)

Show bounds on \( \beta, \gamma \) plane
4. Parametrizations of $V$.

**Standard:**

$$V = CBA$$

$$A = \begin{pmatrix} C_{ij} & S_{ij} & 0 \\ -S_{ij} & C_{ij} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} C_{ij} & 0 & S_{ij} e^{i\delta} \\ 0 & 1 & 0 \\ -S_{ij} e^{i\delta} & 0 & C_{ij} \end{pmatrix}, \quad C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & C_{ij} & S_{ij} \\ 0 & -S_{ij} & C_{ij} \end{pmatrix}$$

$C_{ij} = \cos \theta_{ij}$, $S_{ij} = \sin \theta_{ij}$ with $\theta_{ij}$ in 1st quadrant.

**Wolfgang:**

$$S_{12} = \lambda, \quad S_{23} = A \lambda \quad S_{13} e^{i\delta} = A \lambda^2 (\rho + i \eta) = \frac{A \lambda^2 (\rho + i \eta) \sqrt{1 - A^2 \lambda^2}}{\sqrt{1 - \lambda^2} (1 - A^2 \lambda^2 \rho + i \eta)}$$

**Exercise (ii) Show that**

$$\hat{p} + i \hat{q} = \frac{V_{\rho} \psi_{\varphi}}{V_{\varphi} \psi_{\rho}} \quad \text{hence phase invariant}.$$  

(1a) **Expand in $\lambda \ll 1$ to show**

$$V = \begin{pmatrix} 1 - \frac{1}{2} \lambda^2 & \lambda & A \lambda^3 (\rho + i \eta) \\ -\lambda & 1 - \frac{1}{2} \lambda^2 & A \lambda^3 \\ A \lambda^3 (1 - \rho + i \eta) & -A \lambda^3 & 1 \end{pmatrix} + \mathcal{O}(\lambda^n)$$
Back to flavor symmetry: $U(3)^3$

Suppose we extend the SM by adding terms (local, nontrivial gauge inv.)
dim $Y$ operators that are invariant under $U(3)^3$ — including

spurions $\lambda u, \lambda d$. For example

$$\Delta \mathcal{L} = \sum_i \mathcal{O}_i,$$

with

$$\mathcal{O}_i = \theta_i \bar{u}_i \lambda \hat{U}_i^\dagger \lambda \bar{q}_j \bar{q}_j,$$

$$\mathcal{O}_2 = \omega \bar{u}_i \lambda \bar{u}_i \lambda \bar{d}_j \bar{d}_j \lambda \bar{d}_j \bar{d}_j$$

Now, go to basis with diagonal mass matrices:

$$\mathcal{O}_1 \rightarrow G_\omega \hat{U}_i \lambda \hat{U}_i^\dagger \lambda \left[ \begin{array}{c} U_{u_i} U_{u_j} \\ U_{d_i} U_{d_j} \end{array} \right]$$

$$= G_\omega \hat{U}_i \lambda \hat{U}_i^\dagger \lambda \left[ \begin{array}{c} U_{u_i} U_{u_j} \\ U_{d_i} U_{d_j} \end{array} \right]$$

The only off-diagonal interaction (in flavor) is determined by $X u V$

Similarly, $\mathcal{O}_2 \rightarrow \tilde{q}' \tilde{u}_i \lambda \bar{u}_i \lambda \tilde{q}' \tilde{d}_j \bar{d}_j \lambda \bar{d}_j \bar{d}_j$, where $\tilde{q}' = \left( \begin{array}{c} U_{d_i} \\ V_{d_i} \end{array} \right)$

Of course $X u V = X u V$, so $X u V = (X u V)^T$

Exercise: show this is generally true, i.e., flavor change is determined by $V$, or more specifically $X u V$ or $X d V$. (or $U X u V$ or $X d V$).

This is the principle of Minimal Flavor Violation (MFV).

Extension of the SM in which the only breaking of $U(3)^3$ is by $\lambda u$ and $\lambda d$

automatically satisfy MFV. As we will see they are least constrained

by flavor changing and CPV observables.
Examples:

1. **SUSY - SM.** In the absence of SUSY this is \( \text{MFV:} \)

\[
\mathcal{L} = \int d^4 \theta \left[ \bar{Q} e^v Q + \bar{U} e^v U + \bar{D} e^v D \right] + \text{gauge kinetic terms (} \frac{1}{2} \partial^a W \partial_a^* W \text{)} + \text{h.c.} \\
+ \int d^2 \theta W + \text{h.c.}
\]

with \( W = H_1 U D \), \( Q + H_2 \lambda \), \( \text{quark terms} \)

Here \( Q, U, D \) are superfields with

\[
\begin{align*}
Q & \sim (3, 2)_Y \\
U & \sim (\overline{3}, 1)_Y \\
D & \sim (\overline{3}, 1)_Y
\end{align*}
\]

Add soft SUSY breaking

\[
\delta \mathcal{L} = \delta \phi^*_q M^2_q \phi_q + \delta \phi^*_u M^2_u \phi_u + \delta \phi^*_d M^2_d \phi_d \\
+ \left( \delta \phi^*_u q_d \phi_d + \delta \phi^*_d q_d \phi_d + \text{h.c.} \right)
\]

Unless \( M^2_q \propto \Lambda \) and \( g_{su} \propto \Lambda \)

^2 new flavor changing interactions

are present and large. The effect can be made small if the masses

of scales (from diagonalizing \( u^2 \mathcal{M}_q^2 + M^2_u \mathcal{V} + \mathcal{V} \mathcal{M}_d \mathcal{V}^* + \Lambda \quad \ldots \) have large eigenvalues.

This is the motivation for gauge-mediated SUSY

\begin{align*}
\text{Gauge sector} & \quad \overset{\text{Gauge mediation}}{\rightarrow} \quad \text{SUSY sector} \\
\end{align*}

Gauge interactions in \( \bar{Q} e^v Q \) are flavorful

(\text{In SU(1) modulated the problem is severe).}
Note that for $U(3)^3$ to be a symmetry, the squarks must transform
too, just like quarks: $q_i \to U_{ij} q_j$, $\phi_i \to U_{ij} \phi_j$, etc.

- **MFV fields**: Recently $t \bar{t} F B$ asymmetry, possibly explained by

1. $s$-channel, e.g., axigluon: 
   $$ \frac{1}{\sqrt{2}} \frac{g}{g} \tilde{t} \tilde{b} + \frac{1}{\sqrt{2}} \frac{g}{g} \tilde{b} \tilde{t} $$

2. $t$-channel, e.g., scalar: 
   $$ \frac{1}{\sqrt{2}} \frac{g}{g} \tilde{t} \tilde{b} + \frac{1}{\sqrt{2}} \frac{g}{g} \tilde{b} \tilde{t} $$

I won't explain why axigluon involves $U(6)^3$ breaking (roughly, one needs
opposite sign couplings of $\tilde{t} \tilde{b} \tilde{t} \tilde{b}$ and $\tilde{b} \tilde{t} \tilde{b} \tilde{t}$).

Concentrate on $t$-channel models. Clearly $\phi \bar{t} \bar{u}$ breaks $U(3)^3$.

Unless one has extreme fine tuning, one will also have $2 \bar{t} \tilde{b} \tilde{t} \tilde{b}$ couplings,
and if $L$-handed quarks are involved, also $\tilde{b}_L \tilde{b}_L \bar{t}_L \bar{t}_L$ couplings.

One can have a $U(3)^3$-symmetric model by including a scalar multiplet
that transforms under $U(3)^3$. For example, one can have

$$ \tilde{q}_L \phi \bar{u}_R \text{ with } \phi \to U_{ij} \phi U^\dagger_{jk} \text{ (and a } 2 \text{ under } SU(3)_L \times SU(3)_R).$$

This actually works (see:

**Exercise**: classify all possible dim-4 interactions $\phi \bar{t} \bar{u} \phi'$
and corresponding transformation laws for $\phi$ (under $U(3)^3 \times SM$-gauge):

1. $U(3)^3 \times U(3)$, $U(3)$ with up to $\mathcal{O}_{\phi \bar{t} \bar{u} \phi'}$. 

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Lecture 1 (9/14)
FCNC's

stands for Flavor Changing Neutral Currents

but is used more generally to mean FCN-transitions.

FC transitions in SM:

1. Tree level. Only $W^3$:

$$n \rightarrow p e \bar{v}$$

Exercise: If you have never computed $\mu$-(lifetime), check that

$$\tau(\mu \rightarrow e \bar{\nu} \bar{\nu}) = \frac{G_F m_{\mu}^5}{192 \pi^3} \quad \text{for } m_{\mu} = 0$$

But 2nd interactions are diagonal in flavor

eg

$$U \begin{bmatrix} d_L \nonumber \\ u_L \nonumber \end{bmatrix}, \begin{bmatrix} d_R \nonumber \\ u_R \nonumber \end{bmatrix}, \begin{bmatrix} u_L \nonumber \\ d_L \nonumber \end{bmatrix} \begin{bmatrix} s_L \nonumber \\ c_L \nonumber \end{bmatrix}, \begin{bmatrix} s_R \nonumber \\ c_R \nonumber \end{bmatrix}$$

2. 1-loop: Can we have FCNC's at 1-loop? Say $b \rightarrow s \gamma$?

$$b \rightarrow W^+ \gamma \rightarrow s$$

$\Rightarrow$ FCNC's are suppressed in SM relative to tree level

by $\sim \frac{g^2}{16\pi^2} \sim \frac{\alpha}{4\pi} \frac{G_F^2}{\Lambda^2}$
GIM mechanism: more suppression of FCNC in SM

(1) “Old” world: imagine a world with $m_u < m_c < m_t < m_w$ 
(Real world $m_\theta < m_c < m_t < m_w \approx 1/2 m_t$).

In most explicit computation of integrals, we use that

$$b \left( \frac{m_w}{m} \right)^s = e q \epsilon_u \left( \frac{\bar{u}(p) \sigma^{\mu\nu} \left( \frac{i}{2} \gamma \right) u(p) m_w^2}{M_w^2} \right) \frac{g_s^2}{16\pi^2} \cdot I$$

where $I = \sum_{i=0} V_{ib} V_{is}^* \frac{F\left( \frac{m_i^2}{m_w^2} \right)}{M_w^2}$

Now, expand in Taylor series $F(x) = F(0) + x F'(0) + ...$

and use $\sum_{i=0} V_{ib} V_{is}^* = 0$

$$I = \left( \sum_{i=0} \frac{F(0)}{i!} \frac{V_{ib} V_{is}^*}{M_w^2} \right) F(0) + \left( \sum_{i=0} \frac{F'(0)}{i!} \frac{V_{ib} V_{is}^*}{M_w^2} \right) F'(0) + ...$$

Moving over $\sum_{i=0} V_{ib} V_{is}^* = -V_{ib} V_{is}^*$

so $I = F'(0) \sum_{i=0} \frac{V_{ib} V_{is}^*}{M_w^2} \frac{m_i^2 - m_b^2}{M_w^2}$

$\Rightarrow$ The FCNC is suppressed in addition to the loop, by

$$\sim V_{ub} V_{us}^* \frac{m_c^2 - m_t^2}{M_w^2} + V_{tb} V_{ts}^* \frac{m_b^2 - m_t^2}{M_w^2} \sim \epsilon^2 \frac{m_t^2}{M_w^2} + \epsilon^2 \frac{m_t^2}{M_w^2}$$

That is, both by $\frac{m_t^2}{M_w^2}$ and by $\epsilon^2$. 

"Modern" GIM

Of course, $m_u < m_w$ is not a good approximation. But the suppression by CKM's $c^2$ is still there:

$$1 = \sum_n V_{ub}V_{ub}^* F\left(\frac{m_u^2}{m_w^2}\right) = -\sum_n V_{ub}V_{ub}^* \left(\frac{F\left(\frac{m_t^2}{m_w^2}\right)}{m_w^2} - \frac{m_u^2}{m_w^2}\right) \approx c^2 \left(\frac{F\left(\frac{m_t^2}{m_w^2}\right)}{m_w^2} - \frac{m_u^2}{m_w^2}\right)$$

It turns out that $F(x)$ is an increasing function with $F(1) = 0 \Rightarrow m_w^2$ is unimportant.

$\Rightarrow$ the virtual top-quark exchange dominates this amplitude.

Exercise: show that for $s \approx d$ it is no longer true that

virtual top-quark dominates, that in fact $c \theta$ contributions are numerically (roughly) the same magnitude.
Bounds on \( N \bar{P} \), push.

\[ \Delta f = \frac{e F_{\nu} H^{2}_{\nu} \sigma_{\nu}^{2} \sqrt{S_{\nu}}}{\sqrt{2} \mu} \rightarrow \frac{e}{\sqrt{2} \mu} T_{\nu} \sigma_{\nu}^{2} S_{\nu} \]

So, roughly,

\[ \frac{\Delta f}{\Delta m_{\nu}} = \frac{1}{\sqrt{2} \mu} \frac{e \sigma_{\nu}^{2} \sqrt{S_{\nu}}}{1/2} \frac{W_{\nu}^{2}}{g_{\nu}^{2}} \]

and requires \( \Delta m_{\nu} \leq 10\%

(since the SM predicts agreement with experiment)

\[ \Lambda \geq \frac{\sqrt{M_{\nu}^{2}}}{\sqrt{	ext{det}(V_{\nu})}} \left( \frac{1}{2} \right) \rightarrow \Lambda > 70 \text{ TeV} \]

With MFV:

\[ \Delta f = \frac{e F_{\nu} H^{2}_{\nu} \sigma_{\nu}^{2} \sqrt{S_{\nu}}}{\sqrt{2} \mu} \rightarrow 0 \]

So again but

\[ \Lambda \geq \frac{\Lambda_{\nu}^{7} \lambda_{\nu}^{4} \lambda_{\nu}^{4} \lambda_{\nu}^{8}}{g_{\nu}^{2} \mu_{\nu}^{2}} \rightarrow \Lambda > 4 \text{ TeV} \]

Now

\[ \frac{\Delta f}{\Delta m_{\nu}} \sim \frac{\sqrt{M_{\nu}^{2}}}{\sqrt{2} \mu} \frac{1}{g_{\nu}^{2} \mu_{\nu}^{2}} \]

Then, \( \Lambda > 4 \text{ TeV} \)
Determination of CKM / U.T.

**Magnitudes:**

(i) $V_{ud}$ (nuke),

(ii) $V_{us}, V_{cd}, V_{cs}, V_{ub}, V_{cb}$, $M \rightarrow M' + \nu$ (e.g., $K \rightarrow \pi^0 e^+ e^-$)

\[
\langle p' | V^a | p \rangle = \sum_{g}(g^a)^r \epsilon_{r}(g^{a})_{1}^{+} + \sum_{g}(g^{a})_{2}^{+} q \rightarrow q = (p - p')^r
\]

\[
M = \bar{p} (\text{or } p' \text{ by } P_{M} = p' r)
\]

Central problem: $f_+$.

\rightarrow Symmetry: for same state $(q_{-})\partial_v = 0 \rightarrow f_{-} = 0$ and $f_{+}(0) = 1$

Fr. $K \rightarrow \pi^0$ (SU(3) symmetry, $f_{+}(0) = 1 + O(M^2)$)

Ademollo-Gatto-Theo.

\[
\bar{B} \rightarrow D\bar{K} (b \rightarrow c\bar{u}) : \quad \text{HQ symmetry (some up with H G T):}
\]

The "brown much b" is in both cases bound by an oo-heavy quark source of color.

In some sense $f_{+}(0) = 1$ here too.

Long reliance: get rid of $m \rightarrow 0$ to take much limit, eg, use $\nu \rightarrow \nu_m$

and normalize states to $1$ (as in NR physics).

Then $\langle 0 | \bar{D} | B(0) \rangle = \frac{1}{2} \nu | 0 \rangle | \nu + \nu \rangle \quad \bar{\nu}(0) = 1$ is the Isgur-Wise \( f_{+} \) function

(iii) $X^{u-K}\text{ mixing}$: we'll use $u = u_{d}v_{ud}^{*}$, $d = v_{ud}^{*}v_{ub}^{*}$

(iv) $\nu, \bar{\nu}$ are (lo) directly from CPV asymmetries

(vi) Study $\sin(2\beta)$ - the prime proxy of more