Jets

When low scales $s - \Lambda_{QCD}$ are reached in the shower, QCD will bind the partons into hadrons. Sprays of hadrons form the jets observed experimentally.

Specify a *jet algorithm* for combining the observed particles into jets.

The idea: the jets should reflect the primordial hard partons.
When low scales $s - \Lambda_{\text{QCD}}$ are reached in the shower, QCD will bind the partons into hadrons. Sprays of hadrons form the jets observed experimentally.

Specify a jet algorithm for combining the observed particles into jets.

Several important properties that should be met by a jet definition are [3]:

1. Simple to implement in an experimental analysis;
2. Simple to implement in the theoretical calculation;
3. Defined at any order of perturbation theory;
4. Yields finite cross sections at any order of perturbation theory;
5. Yields a cross section that is relatively insensitive to hadronisation.

Useful reference: G. Salam, 0906.1833
The cone

Basic idea: draw a cone around the clusters of energy in the event

**Iterated cones:**
- Start with seed particle $i$
- Combine all particles within a cone of radius $R$
- Use the combined 4-momentum as a new seed
- Repeat until stability achieved

\[ \Delta R_{ij}^2 = (y_i - y_j)^2 + (\phi_i - \phi_j)^2 < R^2 \]

Example:
- Progressive removal (IC-PR): start with largest transverse momentum as seed; after finding stable cone, call it a jet and remove; go to next largest $p_T$
We saw before that IR singularities cancel between real, virtual corrections $\Rightarrow$ *infrared safety*. The jet algorithm shouldn’t spoil this cancellation. The example on the previous slide does.

IC-PR algorithm starts from different seed after emission of a hard collinear parton

Infinities do not cancel
Consequences

- Consequence: \( \frac{1}{\epsilon} \rightarrow \ln(p_T/\Lambda_{\text{QCD}}) \sim 1/\alpha_S \) ⇒ no suppression of higher-order contributions, no expansion possible

\[
\begin{align*}
\alpha_s \alpha_{\text{EW}} + \alpha_s^2 \alpha_{\text{EW}} + \alpha_s^3 \alpha_{\text{EW}} \ln \frac{p_T}{\Lambda} + \alpha_s^4 \alpha_{\text{EW}} \ln^2 \frac{p_T}{\Lambda} + \cdots,
\end{align*}
\]

\[
\sim \alpha_s \alpha_{\text{EW}} + \alpha_s^2 \alpha_{\text{EW}} + \alpha_s^2 \alpha_{\text{EW}} + \alpha_s^2 \alpha_{\text{EW}} + \cdots
\]

<table>
<thead>
<tr>
<th>Observable</th>
<th>1st miss cones at</th>
<th>Last meaningful order</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inclusive jet cross section</td>
<td>NNLO</td>
<td>NLO</td>
</tr>
<tr>
<td>( W/Z/H + 1 ) jet cross section</td>
<td>NNLO</td>
<td>NLO</td>
</tr>
<tr>
<td>3 jet cross section</td>
<td>NLO</td>
<td>LO</td>
</tr>
<tr>
<td>( W/Z/H + 2 ) jet cross section</td>
<td>NLO</td>
<td>LO</td>
</tr>
<tr>
<td>jet masses in 3 jets, ( W/Z/H + 2 ) jets</td>
<td>LO</td>
<td>none</td>
</tr>
</tbody>
</table>

Situation for midpoint cone, from Salam & Soyez 0704.0292

- Can modify algorithms so that addition of soft/collinear particles doesn’t modify hard jets in the event: SIScone (seedless infrared safe)
Sequential recombination

- **$k_t$ algorithm:**
  
  \[
  d_{ij} = \min(p_{ti}^2, p_{tj}^2) \frac{\Delta R_{ij}^2}{R^2}
  \]
  
  \[
  d_{iB} = p_{ti}^2
  \]

  - Work out all $d_{ij}$, $d_{iB}$, find minimum
  - If it is a $d_{ij}$, combine i and j and restart
  - If it is a $d_{iB}$, call i a jet and remove it
  - Stop after no particles remain

- **Generalizations use a slightly different distance measure**
  
  \[
  d_{ij} = \min(p_{ti}^{2p}, p_{tj}^{2p}) \frac{\Delta R_{ij}^2}{R^2}
  \]
  
  \[
  d_{iB} = p_{ti}^{2p}
  \]

  - $p=-1$: anti-$k_t$
  - $p=0$: Cambridge-Aachen

- **Roughly, soft and collinear emissions come with small distance measure and are always recombined \( \Rightarrow \) IR safe**
Areas denote where soft radiation would be “soaked up” by jet

First clusters all sorts of soft particles, which eventually become added to jet; more sensitive to underlying event, pile-up

Avoids this with the $1/p_t^2$ in $d_{ij}$; the preferred choice for LHC studies
Recent interest in using substructure of jets to distinguish signal from background. For example, highly-boosted Higgs will produce a “fat jet” with two $b$ subjets inside.

Undo last stage of clustering and look for significant mass drop, consistent with heavy particle decaying to jets.

Boosted tops, $W/Z$ bosons have been studied in various contexts.

Butterworth et al., 0802.2470
Example 3: Deep inelastic scattering and PDFs
Deep inelastic scattering

Putting one hadron in the initial state leads to DIS ⇒ still gives most of our information on PDFs (ep at DESY)

Kinematics:

\[ q^\mu = k'^\mu - k^\mu \]
\[ Q^2 = -q^2 \]
\[ x = \frac{Q^2}{2P \cdot q} \]
\[ y = \frac{P \cdot q}{P \cdot k} \]

\[ d\sigma = \frac{4\alpha^2 d^3k'}{s} \frac{1}{2|k'| Q^4} L^{\mu\nu}(k,q) W_{\mu\nu}(p,q) \]

- phase space scat. lepton
- photon propagator
- leptonic tensor
- hadronic tensor
- contains information about hadronic structure
Hadronic tensor

Hermiticity, parity, current conservation allow us to simplify $W_{\mu\nu}$

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4z \ e^{iq \cdot z} \langle P | J_\nu^+(z) J_\mu(0) | P \rangle$$

$$= \left\{ g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right\} F_1(x, Q^2) + \left\{ P_\mu + \frac{q_\mu}{2x} \right\} \left\{ P_\nu + \frac{q_\nu}{2x} \right\} \frac{F_2(x, Q^2)}{P \cdot q}$$

**Structure functions**

$$\frac{d\sigma}{dx \ dQ^2} = \frac{4\pi \alpha^2}{Q^4} \left\{ [1 + (1 - y)^2] \ F_1 + \frac{1 - y}{x} \ [F_2 - 2x \ F_1] \right\}$$

**Factorization.** tells us that EM probe scatters off partons

$$W_{\mu\nu} = \frac{1}{4\pi} \int d^4z \ e^{iq \cdot z} \int_0^1 \frac{d\xi}{\xi} \ \sum_a f_a(\xi) \langle p | J_\nu^+(z) J_\mu(0) | p \rangle_{p=\xi P}$$

**PDFs**
Calculating the structure function

We will calculate the structure function $F_2$. Note that we can obtain it by applying the following projection operator to $W$:

$$F_2 = R^\mu_\nu W_{\mu_\nu}$$

$$R^\mu_\nu = \frac{2x}{d-2} \left\{ g^\mu_\nu - 4 (d-1) \frac{x^2}{Q^2} P^\mu P^\nu \right\}$$

Calculate by inserting a complete set of states between currents; at LO, have a single-quark final state:

Parameterize momenta as:

$$P^\mu = \frac{Q}{2x} \left( 1, \vec{0}, 1 \right)$$

$$p_i^\mu = p^\mu = \frac{\xi Q}{2x} \left( 1, \vec{0}, 1 \right)$$

$$q^\mu = \left( 0, \vec{0}, -Q \right)$$
Calculating the structure function

We will calculate the structure function $F_2$. Note that we can obtain it by applying the following projection operator to $W$:

\[
F_2 = R^{\mu\nu} W_{\mu\nu}
\]

\[
R^{\mu\nu} = \frac{2x}{d-2} \left\{ g^{\mu\nu} - 4(d-1) \frac{x^2}{Q^2} P^\mu P^\nu \right\}
\]

Calculate by inserting a complete set of states between currents; at LO, have a single-quark final state:

\[
PS = \int \frac{d^d p_f}{(2\pi)^{d-1}} \delta(p_f^2)(2\pi)^d \delta^{(d)}(q + p - p_f)
\]

\[
= \frac{2\pi}{Q^2} \delta \left( 1 - \frac{x}{\xi} \right)
\]

Note that the virtual corrections will have the same phase-space; will be important later.
Calculating the structure function

We will calculate the structure function $F_2$. Note that we can obtain it by applying the following projection operator to $W$:

$$ F_2 = R^\mu\nu W_{\mu\nu} $$

$$ R^{\mu\nu} = \frac{2x}{d-2} \left\{ g^{\mu\nu} - 4(d-1) \frac{x^2}{Q^2} P^\mu P^\nu \right\} $$

Calculate by inserting a complete set of states between currents; at LO, have a single-quark final state:

Obtain the structure function:

$$ F_2 = \frac{1}{4\pi} \int \frac{d\xi}{\xi} \sum_q f_q(\xi) \times \frac{PS}{2N} \times R^{\mu\nu} \times W_{\mu\nu} $$

$$ = \sum_q e^2 Q_q^2 \int d\xi f_q(\xi) \xi \delta(x-\xi) $$

$$ = \sum_q e^2 Q_q^2 x f_q(x) $$
Scaling

No $Q^2$ dependence in $F_2 \Rightarrow$ scaling, comes from scattering off point-like constituents of proton.

- Clearly a good approximation, but also clearly violated.
- Goal: check to see that QCD reproduces the scaling violation.
- Possible NLO real-emission terms:

\[
\Rightarrow \text{we'll do the quark pieces and quote the answer for these}
\]
Real-emission phase space

Focus on new aspects with respect to $e^+e^- \rightarrow$ hadrons; first, derive a useful parameterization of the phase space

$$PS = \frac{1}{(2\pi)^{d-2}} \int d^dp_f d^dp_g \delta(p_g^2) \delta(p_f^2) \delta^{(d)}(q + p - p_f - p_g)$$

$$= \frac{1}{(2\pi)^{d-2}} \int ds_{pg} \int d^dp_f d^dp_g \delta(p_g^2) \delta(p_f^2) \delta(s_{pg} + 2p \cdot p_g) \delta^{(d)}(q + p - p_f - p_g)$$

Parameterize $p_g$ as: $p_g = (E, p_T, 0, k)$; use delta functions to remove these integrations.

Set $s_{pg} = Q^2 \xi z / x$, which defines $z$, to derive:

$$PS = \frac{\Omega(d-2)}{4(2\pi)^{d-2}} \int_0^1 dz \left[ Q^2 z (1 - z) \frac{\xi}{x} \left( 1 - \frac{x}{\xi} \right) \right]^{-\epsilon}$$

$$p \cdot p_g = \frac{\xi}{2x} Q^2 z$$

$$p_f \cdot p_g = \frac{\xi}{2x} Q^2 \left( 1 - \frac{x}{\xi} \right)$$
Real-emission matrix elements

- Spin, color summed/averaged+projected matrix elements; focus on the potentially divergent terms

\[ |\mathcal{M}|^2 = 4 C_F e^2 Q^2 g_s^2 \mu^2 e \left\{ \frac{p_f \cdot p_g}{p \cdot p_g} + \frac{p \cdot p_g}{p_f \cdot p_g} + \frac{Q^2 p \cdot p_f}{p_f \cdot p_g p \cdot p_g} + \cdots \right\} \]

- Need to integrate over \( y \), include

\[ \frac{1}{4\pi} \int \frac{d\xi}{\xi} f_q(\xi) \]

- \( F_{2,q}^{(1),real} = e^2 Q^2 x \frac{\alpha_s}{2\pi} \frac{1}{\Gamma(1-\epsilon)} \left[ \frac{Q^2}{4\pi \mu^2} \right]^{-\epsilon} \int_x^1 \frac{d\xi}{\xi} f_q(\xi) \]

\[ \times \left( \frac{x}{\xi} \right)^\epsilon \left( 1 - \frac{x}{\xi} \right)^{-\epsilon} \left\{ -\frac{C_F}{\epsilon} \frac{1 + (x/\xi)^2}{1 - x/\xi} - 2C_F \frac{x/\xi}{1 - x/\xi} + \cdots \right\} \]

This term is bad news, no way it can cancel against virtual correction, which are \( \delta(x-\xi) \)

Looks like \( P_{qq} \Rightarrow \) collinear singularity

Notice the singularity when \( x=\xi \Rightarrow \) soft singularity
Factorization of IR singularities

- We are not satisfying the KLN theorem, which tells us to sum over degenerate final and initial states. The quark from the proton can emit a collinear gluon. This changes the momentum of the quark that enters the partonic scattering process, but is indistinguishable. The virtuality associated with this splitting is very small, and this is a long-distance effect sensitive to low-energy QCD.

- Solution: must absorb initial-state collinear singularity into PDF. Redo calculation with $f_q \rightarrow f_{q,0}$, a bare PDF. Choose the bare PDF to remove $1/\epsilon$ pole.

- Must also add virtual corrections, deal with the $x=\xi$ soft singularity of real emission.
We will perform this ‘mass factorization’ step-by-step. First define a plus distribution:

\[ \int_0^1 dx f(x) [g(x)]_+ = \int_0^1 dx g(x) [f(x) - f(0)] \Rightarrow \text{if } g \sim 1/x, \text{ removes singularity at } x=0 \]

After adding virtual corrections and rearranging, our result for the divergent part of \( F_2 \) is:

\[
F_{2,q} = e^2 Q_q^2 x \int_x^1 \frac{d\xi}{\xi} f_{q,0}(\xi) \left\{ \delta(1 - x/\xi) + \frac{\alpha_s}{2\pi \Gamma(1 - \epsilon)} \left[ \frac{Q^2}{4\pi \mu^2} \right]^{-\epsilon} \left[ -\frac{1}{\epsilon} P_{qq}(x/\xi) + \text{finite} \right] \right\}
\]

\[
P_{qq}(x) = C_F \left[ \frac{1 + x^2}{[1 - x]_+} + \frac{3}{2} \delta(1 - x) \right] \left( \Rightarrow \int_0^1 P_{qq}(x) = 0 \right) \quad \text{quark-number conservation}
\]
Factorization of IR singularities

We will perform this ‘mass factorization’; step-by-step. First define a plus distribution:

\[
\int_0^1 dx f(x) [g(x)]_+ = \int_0^1 dx g(x) [f(x) - f(0)] \quad \Rightarrow \text{if } g \sim 1/x, \text{ removes singularity at } x=0
\]

Redefine the PDF according to:

\[
f_q(x, \mu^2) = f_{q,0}(x) + \frac{\alpha_s}{2\pi} \int_0^1 \frac{d\xi}{\xi} f_{q,0}(\xi) \left\{ -\frac{1}{\epsilon} P_{qq}(x/\xi) + C(x/\xi) \right\}
\]

\[\text{MS: C chosen to remove } \ln(4\pi) - \gamma_E\]

Arrive at the structure function:

\[
F_{2,q} = e^2 Q_q^2 x \int_x^1 \frac{d\xi}{\xi} f_q(\xi, \mu^2) \left\{ \delta(1 - x/\xi) + \frac{\alpha_s}{2\pi} \left[ P_{qq}(x/\xi) \ln \frac{Q^2}{\mu^2} + \text{finite} \right] \right\}
\]
Scale variation and DGLAP

Pole turns into a $\ln(\mu^2)$ dependence $\Rightarrow$ $F_2$ must be independent of this arbitrary factorization scale, which leads to an evolution equation for the PDF. Renormalization $\Rightarrow$ Evolution.

$$\frac{d f_q(x, \mu^2)}{d \ln \mu^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} f_q(\xi, \mu^2) P_{qq}(x/\xi) \quad \Rightarrow \text{DGLAP equation.}$$

$\square$ Leads to a $\ln(Q^2)$ dependence of $F_2$ $\Rightarrow$ explains the observed scaling violation

Inclusion of the gluon-initiated partonic processes:

$$F_{2,q} = e^2 Q^2 q x \int_x^1 \frac{d\xi}{\xi} f_q(\xi, \mu^2) \left\{ \delta(1 - x/\xi) + \frac{\alpha_s}{2\pi} \left[ P_{qq}(x/\xi) \ln \frac{Q^2}{\mu^2} + \text{finite} \right] \right\}$$

$$+ e^2 Q^2 q x \int_x^1 \frac{d\xi}{\xi} f_g(\xi, \mu^2) \left\{ \frac{\alpha_s}{2\pi} \left[ P_{qq}(x/\xi) \ln \frac{Q^2}{\mu^2} + \text{finite} \right] \right\}$$

$$\frac{d}{d \ln \mu^2} \left( \begin{array}{c} f_q(x, \mu^2) \\ f_g(x, \mu^2) \end{array} \right) = \frac{\alpha_s}{2\pi} \int_x^1 \frac{d\xi}{\xi} \left( \begin{array}{ccc} P_{qq}(x/\xi) & P_{pq}(x/\xi) & f_q(x, \mu^2) \\ P_{gq}(x/\xi) & P_{gg}(x/\xi) & f_g(x, \mu^2) \end{array} \right)$$
PDFs

- Get much of our knowledge of PDFs from the DIS process
- PDFs enter every hadron collider prediction, so we’d better know them well. Non-perturbative objects with perturbative evolution. $f(x, Q^2)$: DGLAP governs $Q^2$ dependence, so we need to extract the $x$ dependence from data.
- On the market today: CTEQ, MSTW, NNPDF (global fits) ABM, HERAPDF, JR (non-global)
- Basic idea:

  hadronic cross section = PDFs $\otimes$ partonic cross section

  measure \hspace{1cm} extract \hspace{1cm} calculate
Determining PDFs

In more detail (from the *Handbook of Perturbative QCD*):

1. Develop a program to numerically solve the evolution equations — a set of coupled integro-differential equations;

2. Make a choice on experimental data sets, such that the data can give the best constraints on the parton distributions;

3. Select the factorization scheme — the “DIS” or the “$\overline{\text{MS}}$” scheme, and make a consistent set of choices on factorization scale for all the processes;

4. Choose the parametric form for the input parton distributions at $\mu_0$, and then evolve the distributions to any other values of $\mu_f$;

5. Use the evolved distributions to calculate $\chi^2$ between theory and data, and choose an algorithm to minimize the $\chi^2$ by adjusting the parameterizations of the input distributions;

6. Parameterize the final parton distributions at discrete values of $x$ and $\mu_f$ by some analytical functions.
Global fits typically use HERA charged and neutral current data; fixed-target Drell-Yan and DIS; jet production from the Tevatron/LHC; W/Z data from the Tevatron/LHC

Non-global fits typically remove one or more of these for various reasons; for example, ABM neglects jet production, since it’s not known at NNLO in pQCD

- fixed-target DY and DIS
- HERA
- Tevatron
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• Non-global fits typically remove one or more of these for various reasons; for example, ABM neglects jet production, since it’s not known at NNLO in pQCD

\[\text{Need this large multiplicity to get all partons across the needed range of } x\]

<table>
<thead>
<tr>
<th>Process</th>
<th>Subprocess</th>
<th>Partons</th>
<th>$x$ range</th>
</tr>
</thead>
<tbody>
<tr>
<td>$e^\pm {p, n} \to e^\pm X$</td>
<td>$\gamma^* q \to q$</td>
<td>$g, q, g$</td>
<td>$x \geq 0.01$</td>
</tr>
<tr>
<td>$\ell^\pm n/p \to \ell^\pm X$</td>
<td>$\gamma^* d/u \to d/u$</td>
<td>$d/u$</td>
<td>$x \geq 0.01$</td>
</tr>
<tr>
<td>$p p \to \mu^+ \mu^- X$</td>
<td>$u\bar{u}, d\bar{d} \to \gamma^*$</td>
<td>$\bar{q}$</td>
<td>$0.015 \leq x \leq 0.35$</td>
</tr>
<tr>
<td>$p n/p p \to \mu^+ \mu^- X$</td>
<td>$(u\bar{d})/(u\bar{u}) \to \gamma^*$</td>
<td>$\bar{d}/\bar{u}$</td>
<td>$0.015 \leq x \leq 0.35$</td>
</tr>
<tr>
<td>$\nu(\bar{\nu}) N \to \mu^- (\mu^+) X$</td>
<td>$W^* q \to q'$</td>
<td>$q, \bar{q}$</td>
<td>$0.01 \leq x \leq 0.5$</td>
</tr>
<tr>
<td>$\nu N \to \mu^- \mu^+ X$</td>
<td>$W^* s \to c$</td>
<td>$s$</td>
<td>$0.01 \leq x \leq 0.2$</td>
</tr>
<tr>
<td>$\nu N \to \mu^+ \mu^- X$</td>
<td>$W^* \bar{s} \to \bar{c}$</td>
<td>$\bar{s}$</td>
<td>$0.01 \leq x \leq 0.2$</td>
</tr>
<tr>
<td>$e^\pm p \to e^\pm X$</td>
<td>$\gamma^* q \to q$</td>
<td>$g, q, \bar{q}$</td>
<td>$0.0001 \leq x \leq 0.1$</td>
</tr>
<tr>
<td>$e^+ p \to \nu X$</td>
<td>$W^+ {d, s} \to {u, c}$</td>
<td>$d, s$</td>
<td>$x \geq 0.01$</td>
</tr>
<tr>
<td>$e^\pm p \to e^\pm c\bar{c} X$</td>
<td>$\gamma^* c \to c, \gamma^* g \to c\bar{c}$</td>
<td>$c, g$</td>
<td>$0.0001 \leq x \leq 0.01$</td>
</tr>
<tr>
<td>$e^\pm p \to \text{jet } X$</td>
<td>$\gamma^* g \to q\bar{q}$</td>
<td>$g$</td>
<td>$0.01 \leq x \leq 0.1$</td>
</tr>
<tr>
<td>$p\bar{p} \to \text{jet } X$</td>
<td>$gg, qg, q\bar{q} \to 2j$</td>
<td>$g, q$</td>
<td>$0.01 \leq x \leq 0.5$</td>
</tr>
<tr>
<td>$p\bar{p} \to (W^\pm \to \ell^\pm \nu) X$</td>
<td>$u d \to W, \bar{u}\bar{d} \to W$</td>
<td>$u, d, \bar{u}, \bar{d}$</td>
<td>$x \geq 0.05$</td>
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<tr>
<td>$p\bar{p} \to (Z \to \ell^+ \ell^-) X$</td>
<td>$u u, d d \to Z$</td>
<td>$u, d$</td>
<td>$x \geq 0.05$</td>
</tr>
</tbody>
</table>
LHC data making an important appearance!
• MS scheme most commonly chosen these days
• Another issue that should appear here: to what order in pQCD are the partonic cross sections calculated?
• All the ones referenced previously (CTEQ, MSTW, NNPDF; ABM, HERAPDF, JR) provide both NLO and NNLO fits
• Note that the NNLO fits of CTEQ, MSTW, NNPDF use NLO QCD predictions for jet production
1. Develop a program to numerically solve the evolution equations — a set of coupled integro-differential equations;
2. Make a choice on experimental data sets, such that the data can give the best constraints on the parton distributions;
3. Select the factorization scheme — the “DIS” or the “$\overline{\text{MS}}$” scheme, and make a consistent set of choices on factorization scale for all the processes;
4. Choose the parametric form for the input parton distributions at $\mu_0$, and then evolve the distributions to any other values of $\mu_f$;
5. Use the evolved distributions to calculate $\chi^2$ between theory and data, and choose an algorithm to minimize the $\chi^2$ by adjusting the parameterizations of the input distributions;
6. Parameterize the final parton distributions at discrete values of $x$ and $\mu_f$ by some analytical functions.

- Traditional choice of CTEQ and MSTW: $f(x, \mu_0) = A_0 x^{A_1} (1-x)^{A_2} P(x)$

from CTEQ: $q_v(x, \mu_0) = q(x, \mu_0) - \bar{q}(x, \mu_0) = a_0 x^{a_1} (1 - x)^{a_2} \exp(a_3 x + a_4 x^2 + a_5 \sqrt{x})$

- NNPDF uses instead a neural network parameterization to remove bias: $f(x, \mu_0) = c(x) \times \text{NN}(x)$
LHC PDFs

Lots of gluons!
Published sets come with errors... what do they mean?

For technical details on how to propagate these errors through to obtain the error on a cross section, see 1101.0536
PDF errors

Published sets come with errors... what do they mean?

- There are many sources of uncertainty in the PDFs, some of which we’ve touched on
  - Data set choice
  - Kinematic cuts
  - Parametrization choices
  - Treatment of heavy quarks, target mass corrections, and higher twist terms
  - Order of perturbation theory
  - Errors on the data ➔ **Only error included!**

- Techniques have been developed to handle the last one
- The others require judgement and experience, but *are not* included in what are generally referred to as PDF errors.

Review by J. Owens at CTEQ 2007 summer school,
http://www.phys.psu.edu/~cteq/schools/summer07/
Some examples meant to recommend caution when interpreting quoted errors

Inclusion of $m_c$, $m_b$ suppresses $F_2$ at low $Q^2 \Rightarrow$ increase $u,d$ to compensate

6-7% increase in LHC W, Z predictions; well outside the quoted error

Note that the estimated uncertainty from higher-order QCD is 1%
PDF error examples

Some examples meant to recommend caution when interpreting quoted errors

MSTW 2008 PDF release arXiv:0901.0002
- Run II inclusive jet data
- Quark-mass effects
- Gluon density decreased at $x \sim 0.1$

$M_{H=170}$ GeV Higgs at Tevatron (pb):

<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>0.3833</td>
<td>0.3988</td>
<td>0.3943</td>
<td>0.3444</td>
</tr>
</tbody>
</table>

$\sim 15\%$ decrease in predicted cross section!
Previous 90% CL error: ±5%
Importance of global fits

- Error estimates from non-global fits must be carefully scrutinized

Example: ABM + JR @ Tevatron

ABM vs MSTW at 160 GeV

-30% (>5 sigma)

Cross section in picobarns

<table>
<thead>
<tr>
<th></th>
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</thead>
<tbody>
<tr>
<td>100</td>
<td>1.438 ± 0.066</td>
<td>1.380 ± 0.076</td>
<td>1.593 ± 0.091</td>
<td>1.682 ± 0.046</td>
<td>1.417</td>
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<tr>
<td>110</td>
<td>1.051 ± 0.052</td>
<td>1.622 ± 0.061</td>
<td>1.209 ± 0.078</td>
<td>1.265 ± 0.038</td>
<td>1.055</td>
</tr>
<tr>
<td>115</td>
<td>0.904 ± 0.047</td>
<td>0.885 ± 0.055</td>
<td>1.060 ± 0.072</td>
<td>1.104 ± 0.034</td>
<td>0.917</td>
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<tr>
<td>120</td>
<td>0.781 ± 0.042</td>
<td>0.770 ± 0.050</td>
<td>0.933 ± 0.067</td>
<td>0.968 ± 0.031</td>
<td>0.800</td>
</tr>
<tr>
<td>125</td>
<td>0.677 ± 0.038</td>
<td>0.672 ± 0.045</td>
<td>0.823 ± 0.062</td>
<td>0.851 ± 0.029</td>
<td>0.700</td>
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<tr>
<td>130</td>
<td>0.588 ± 0.034</td>
<td>0.589 ± 0.041</td>
<td>0.729 ± 0.058</td>
<td>0.752 ± 0.026</td>
<td>0.615</td>
</tr>
<tr>
<td>135</td>
<td>0.513 ± 0.031</td>
<td>0.518 ± 0.037</td>
<td>0.647 ± 0.054</td>
<td>0.666 ± 0.024</td>
<td>0.541</td>
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<tr>
<td>140</td>
<td>0.449 ± 0.028</td>
<td>0.456 ± 0.034</td>
<td>0.576 ± 0.050</td>
<td>0.591 ± 0.022</td>
<td>0.479</td>
</tr>
<tr>
<td>145</td>
<td>0.394 ± 0.025</td>
<td>0.403 ± 0.031</td>
<td>0.514 ± 0.047</td>
<td>0.527 ± 0.020</td>
<td>0.424</td>
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<tr>
<td>150</td>
<td>0.347 ± 0.023</td>
<td>0.358 ± 0.028</td>
<td>0.461 ± 0.044</td>
<td>0.471 ± 0.018</td>
<td>0.377</td>
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<tr>
<td>155</td>
<td>0.306 ± 0.020</td>
<td>0.318 ± 0.026</td>
<td>0.413 ± 0.041</td>
<td>0.421 ± 0.037</td>
<td>0.336</td>
</tr>
<tr>
<td>160</td>
<td>0.271 ± 0.019</td>
<td>0.283 ± 0.024</td>
<td>0.371 ± 0.039</td>
<td>0.378 ± 0.016</td>
<td>0.300</td>
</tr>
</tbody>
</table>

from D. deFlorian
Importance of global fits

- Error estimates from non-global fits must be carefully scrutinized

- Interesting exercise by Thorne and Watt (2011)

  Check how well PDFs reproduce Tevatron jet data

Message from Thorne and Watt: only global analysis provide accurate distributions and uncertainties. No acceptable description of jet data from non-global sets
PDF summary

- Multiple methodologies to cross-check and LHC data gradually increasing robustness of PDF central values and errors
- Global fits in agreement to ~10% or better over entire kinematic range

For more details and all references, see 1101.0536