Effects of D-instantons → particle physics

Instantons associated with Euclidean D2 configurations. The framework is very geometric. CFT techique to write vertex operators.

1. Quantization of physical states in intersecting D-branes

2. Quantization of states with D-instantons: zero modes in this background. We will refer to rigid O(1) instanton (stringy; direct corrections to the holomorphic part of the D-brane action)

3. Applications (calculus with instanton).

4. Prototype
   of O(1) instantons in two directions: multi instantons/U(1)/recombanitations of instantons and U(1) instantons intersecting with D-branes/issues of massive "zero" modes.

Useful references


**Spectrum of intersecting D6-branes (CFT)**

We consider open strings. Open strings attached to the D-brane satisfy Dirichlet boundary conditions transversal to the D-brane and Neumann boundary conditions along the D-brane

\[ \partial_\sigma X^I = 0 \quad \text{at} \quad \sigma = 0, \pi \quad I = 1, \ldots, p + 1 \]

\[ X^J = 0 \quad \text{at} \quad \sigma = 0, \pi \quad J = p + 2, \ldots, 10 \]

where \((\sigma, \tau)\) denote the world-sheet space and time and \(X^I\) the spacetime coordinates.

Worldsheet fermions: NS-\(\psi^I\), R-\(\bar{\psi}^J\). NS sector: Concrete representation of massless states ↔ Vertex operators. Superconformal ghost sector \(\phi = 0\):

\[ V_{A_I}(0) = \xi_I \partial_\sigma X^I e^{ik_1 X^I} \]  

where \(\xi_I\) is the polarization vector in the target space (spin 1 field).

\[ z = e^{\tau E + i\sigma} \]

\[ \bar{z} = e^{\tau E - i\sigma} \]

with \(\text{Im } z > 0\) (only on the upper plane string lives).

Duality trick: We take the whole \(z\)-plane. Superconformal ghost (-1) picture:

\[ V_{A_I}(-1) = \xi_I e^{-\phi} \psi^I e^{ik_1 X^I} \]

\[ V_{A_I}^0 = \xi_I e^{-\phi} \bar{\psi}^J \]  

\(e^{\alpha \Phi}\) has the conformal dimension

\[ [e^{\alpha \Phi}] = -\frac{\alpha(\alpha + 2)}{2} \]  

Similarly

\[ [\psi] = \frac{1}{2} \]

\[ [\partial_\sigma X] = 1 \]

\[ [V_{A_I}] = 1 \]

\[ V_{A_I} = \xi_I e^{-\phi} \psi^I e^{ik_1 x^I} (\Lambda_a \otimes \bar{\Lambda}_b) \]

We have \(N^2\) massless spin fields (different vertex operators). This is in adjoint representation of \(U(N)\).
Orientifold projection

\( \Omega \) action is \( \sigma \rightarrow -\sigma \). There is an action on worldsheet fermions \((-1)^F\). \( \mathbb{Z}_2 \) involution in target space \( \mathcal{M} \) identifies the rigid plane under which the target space has to be invariant. It is so called O-plane. It acts as a D-brane with negative tension.

![Diagram of Orientifold projection](image)

Figure 1:

We have to deal with further projection on the states. For Chan-Paton factors \( \Omega(\Lambda_a \otimes \bar{\Lambda}_b)\Omega^{-1} = -(\Lambda_a \otimes \bar{\Lambda}_b)^T \)

Let us consider string starting on one brane (D6\( _a \)) and ending on the other (D6\( _b \)).

![Diagram of CY x R^3](image)

Figure 2:

At each intersection of cycles we want to quantize the theory. \( \pi_a \circ \pi_b \) is a topological number.

\[
\pi_a \circ \pi_b = N_a^I M_b^I - M_a^I N_b^I \tag{5} \\
\pi_a = N_a^I A^I + M_a^I B_J \tag{6} \\
A^I \circ B_J = \delta^I_J \tag{7} \\
\int_{A_I} \alpha_J = \delta_{IJ} \tag{8}
\]
\[ \int_{B_I} \beta_J = -\delta_{IJ} \quad (9) \]

We have now three Polyakov action, but involved boundary conditions. \( \sigma = \pi \) on \( b \) cycle and \( \sigma = 0 \) on \( a \) cycle. We now involve boundary conditions:

\[
\partial_\sigma X^{2I-1} = X^{2I} = 0 \quad \text{at} \quad \sigma = 0 \\
\partial_\sigma X^{2I-1} + (\tan \pi \theta_I) \partial_\sigma X^{2I} = 0 \quad \text{at} \quad \sigma = \pi \\
X^{2I} - (\tan \pi \theta_I) X^{2I-1} = 0 \quad \text{at} \quad \sigma = \pi
\]

where \( I = 1, 2, 3 \).

Let us go to the complexified notation:

\[
Z^I = X^{2I-1} + iX^{2I} = \sum_{n \in \mathbb{Z}} \frac{\alpha_{n+\theta_I}^I}{n+\theta_I} Z^{-n+\theta_I} + \sum_{n \in \mathbb{Z}} \frac{\bar{\alpha}_{n+\theta_I}^I}{n-\theta_I} \bar{Z}^{-n-\theta_I} 
\]

(10)

Note that \( \bar{\alpha}_{n+\theta_I}^I = \alpha_{n+\theta_I}^I \) and \( \bar{Z}^I = Z^I(\theta_I \rightarrow -\theta_I) \).

We can analogously complexify worldsheet fermions:

\[
\Psi^I = \psi^{2I-1} + i\psi^{2I} = \sum_{r=n+\frac{1}{2}} \tilde{\psi}_{r-\theta_I} z^{-r-\frac{1}{2}+\theta_I} \quad \text{NS} \\
\Psi^I = \psi^{2I-1} + i\psi^{2I} = \sum_{r=n} \tilde{\psi}_{r-\theta_I} z^{-r+\frac{1}{2}+\theta_I} \quad \text{R} 
\]

(11)

(12)

We can identify bosons in NS sector.

Twisted bosonic vacuum \( |\sigma_{\theta_I} \rangle \).

\[
\partial_z Z^I |\sigma_{\theta_I} \rangle \sim z^{-1-\theta_I} |\tau_{\theta_I} \rangle 
\]

(13)

\[
\Psi^I \sim e^{iH_I} \sim e^{i\theta_I H_I} \quad \text{NS} \\
\Psi^I \sim e^{(i+\frac{1}{2})H_I} \sim e^{i(\theta_I-\frac{1}{2})H_I} \quad \text{R}
\]
The last proportionality is due to the twisted boundary conditions.

\[ 0 < \theta_I < 1 \] for \( I = 1, 2 \) and \(-1 < \theta_3 \leq 0\).

\[ V_{-1} = e^{-\phi} \prod_{I=1}^{2} \sigma_{\theta_I} e^{i\theta_I H_I^I} \sigma_{\theta} e^{i(1+\theta_3)H_3} \]

\[ V_{-\frac{1}{2}} = u_{\alpha} e^{\frac{\phi}{2} \sigma_{\theta} e^{i\theta_I H_I^I} \sigma_{\theta} e^{i(1+\theta_3)H_3} \sigma_{\theta} e^{i(\frac{1}{2}+\theta_3)H_3}} \]

with the following conformal weights:

\[
\begin{align*}
[V_{-1}] &= \frac{1}{2} + \sum_{I=1}^{2} \left( \frac{\theta_I (1 - \theta_I)}{2} + \frac{1}{2} \theta_I^2 \right) + \frac{(\theta_3 + 1)(-\theta_3)}{2} + \frac{(\theta_3 + 1)^2}{2} \\
&= \frac{1}{2} + \frac{1}{2} \sum_{I=1}^{3} \theta_I + \frac{1}{2} \frac{\sum_{I=1}^{3} \theta_I}{2} \geq 1 \quad (14) \\
[V_{-\frac{1}{2}}] &= \frac{3}{8} + \frac{1}{8} + 3 \frac{1}{8} = 1 \quad (15)
\end{align*}
\]

when \( \sum_{I=1}^{3} \theta_I = 0 \) bosons become massless. If \( [V_{-1}^0] > 1 \) then \( k_I^2 > 0 \) and so we have tachyon, \( [V_{-1}^0] = 1 \) then \( k_I^2 = 0 \) and these are massless states, and in the case \( [V_{-1}^0] < 1 \) we have \( k_I^2 < 0 \) and these are massive states. \( [V_{-\frac{1}{2}}^1] = 1 \) means that we always have massless spacetime fermions.

Now let us consider a system of \( N_a \) D6\(_a\) and \( N_b \) D6\(_b\) branes.

\[ \pi_a \circ \pi_b = N_{aI} M_{bI}^I - M_{aI}^I N_{bI} > 0 \quad (16) \]

\[ \pi_a \circ \pi_b > 0 \quad (17) \]
In this case we deal with bi-fundamental representation \((\square_a, \square_{a'})\).

In this case we are tensoring two fundamental representations

\[
\square_a \times \square_{a'} = \square_a + \square_{a'}
\]

\[
\square_a = \frac{1}{2} (\pi_{a'} \circ \pi_a + \pi_o \circ \pi_a)
\]

\[
\square_{a'} = \frac{1}{2} (\pi_{a'} \circ \pi_a - \pi_o \circ \pi_a)
\]

Let us put in this context D-instantons.

**D-instantons**

Useful references:


In Type IIA we will deal with Euclidean D2-branes.

\[ \mathbb{R}^{0,1} \times \mathbb{C}Y \]

Figure 7:

Introduce instantons to generate couplings absent in perturbative calculation. They do not preserve \( U(1)_a \). \( E_R, E_L(\underline{a}, \underline{b}) \) couplings are absent since the \( U(1) \) charge is not preserved. And couplings \( (10 \, 10) \, 5_H \) are desired couplings in \( U(5)_{GG} \).

In particular we need neutrino masses to be extremely small. There is a seesaw mechanism to give masses to neutrinos.

\[ h_D \nu H_a L \nu_R \]

Figure 8: \( h_D \nu H_a L \nu_R \) coupling related to \( U(2)_L \times U(1)_a \times U(1)_D \).

It is hard to tune \( h_D \nu \) to be extremely small. Seesaw mechanism introduces direct mass of right-handed neutrinos.

\[
\left( \frac{h_D \nu < H >}{M_R} \right)^2 \sim 10^{-3} \, E\nu \\
M_R \sim 10^{11-15} \, GeV
\]

It is away of explaining neutrino masses in seesaw.
$M_R\nu_R\nu_R$ are perturbatively forbidden. We will not generate them.

If induced by instantons then $M_R$ is

$$M_R \sim e^{-S_E^{cl}/M_sX}$$

where

$$S_E^{cl} = \frac{2\pi}{l_s^3 g_s} \int_{\Xi} \text{Re} (\Omega_3) - i \int_{\Xi} C_3$$

where $\Xi$ is a three cycle wrapped by an instanton. The suppression factor is the volume of that cycle. $S_E^{cl}$ is not invariant under $U(1)$.

$$S_E^{cl} = \frac{2\pi}{l_s^3 g_s} V_{\Xi} = \frac{2\pi}{\alpha} V_{D}$$

where $\alpha$ is the gauge coupling of the brane, $V_D$ is the volume of the cycle wrapped by the brane. If $V_{\Xi}/V_D \sim 1$, then $e^{-S_{cl}} = e^{-\frac{2\pi}{\alpha}}$. Then $M_R \sim 10^{11}$ Gev.

WZ (Wess-Zumino) action associated with particular $D_a$-brane is

$$S_{WZ} \sim \int_{R^3,1} C \wedge e^{F} \sim \int (C_7 + C_5 \wedge F_a + C_3 \wedge F_a \wedge F_a)$$

where $C = C_7 + C_5 + C_3$ with $C_7$, $C_5$ and $C_3$ seven, five and three forms respectively. First term of the integral is the key term for tadpole cancellation, the second one describes gauge bosons associated with $U(1)$ symmetry to be massive.

$$C_5 = C_2^{I}\alpha_1 + D_2^{I}\beta^I$$

$$C_3 = C_0^{I}\alpha_1 + D_{0I}\beta^I$$

From the second term of WZ action we get

$$C_2 \wedge F_a \rightarrow -dC_2 \wedge A_a = -*dC_0 \wedge A_a$$

$$C_2 = N_{aI}C_2^{I} - M_0^{I}D_{2I}$$

$$C_0 = N_{0I}C_0^{I} - M_0^{I}D_{0I}$$

$$C_0 = (C_0^{I}, D_{0I})$$

We expect in 4 dimensions to get the structure of the type $(dC_0 - A_a) \wedge (*dC_0 - *A_a)$. Under a $U(1)$ gauge transformation:

$$A_a \rightarrow A_a + d\Lambda$$

$$C_0^{I} \rightarrow C_0^{I} + (M_0^{I} )\Lambda$$

$$D_{0I} \rightarrow D_{0I} + (N_{aI} )\Lambda$$
If we have orientifolds then
\[
C_2 = (N_a - N'_a)C_2^I - (M_a^I - M'_{aI})D_2I \tag{32}
\]
\[
C_0^I \rightarrow C_0^I + N_a(M_a^I - M'_{aI}) \Lambda \tag{33}
\]
\[
D_{0I} \rightarrow D_{0I} + N_a(N_a - N'_a) \Lambda \tag{34}
\]
if we have \(N_a\) copies.

Axions transform under \(U(1)\), consequently \(\int C_3\) will transform under \(U(1)\) as well. Instantons violate \(U(1)\) charge.

\[
\Xi = N_{\Xi I} A^I + M^I_{\Xi} B_I \tag{35}
\]
\[
C_3 = C_{0I} \alpha^I + D_{0I} \beta^I \tag{36}
\]
\[
\int \Xi = C_0^I N_{\Xi I} - D_{0I} M^I_{\Xi} \tag{37}
\]

Under \(U(1)_a\):

\[
\text{Im}(S_{cl}) \rightarrow (C_0^I N_{\Xi I} - D_{0I} M^I_{\Xi}) + N_a[N_{\Xi I}(M_a^I - M'_{aI}) - M^I_{\Xi}(N_a - N'_a)] \Lambda \tag{38}
\]
\[
e^{-S_{cl}} \rightarrow e^{-S_{cl} + iQ_{\Xi}^a \Lambda} \tag{39}
\]

where

\[
Q_{\Xi}^a = N_a \Xi \circ (\pi_a - \pi'_a) \tag{40}
\]

Instanton action violates \(U(1)\) charge by \(Q_{\Xi}^a\) amount. It can be cancelled by charged fields \(e^{-S_{cl}} \phi^i\).

\[
Q_{\Xi}^a + \sum_i Q_i^a = 0 \quad \forall a \tag{41}
\]

The dominant contributions to \(e^{-S_{cl} - iS_{int}(\mathcal{M},\phi)}\), where \(\mathcal{M}\) are all the zero modes, \(\phi\) is the real matter, come from the minimal volume of the cycle (special Lagrangian)

\[
\int D\mathcal{M} e^{-S_{cl} - iS_{int}(\mathcal{M},\phi)} = S_4^{n.p}(\phi) \tag{42}
\]

Now let us get stringy zero modes. Consider strings starting and ending on the same boundary.

\[
V_{-1} = X^E_\mu e^{-\phi} \psi^\mu \quad NS \tag{43}
\]

From here we have four bosonic zero modes. \(X^E_\mu\) is a polar vector.

In the Ramond sector we have two types of fermionic zero modes associated with breaking down of supersymmetry.

\[
Q^a \bar{Q}^{\dot{a}} \tag{44}
\]
\[
Q^{aI} \bar{Q}^{\dot{a}I} \tag{45}
\]
D-brane on this background breaks 1/2 of supersymmetry and eliminates $Q^\alpha \bar{Q}^{\dot{\alpha}}$ part.

\begin{equation}
V_{-1/2}^\theta = \theta_\alpha e^{-\phi/2} S^\alpha e^{\sum_i \frac{i}{2} H_i}
\end{equation}

\begin{equation}
V_{-1/2}^{\bar{\tau}} = \bar{\tau}_\dot{\alpha} e^{-\phi/2} \bar{S}^{\dot{\alpha}} e^{-i\frac{i}{2} \sum_i H_i}
\end{equation}

\begin{equation}
\frac{1}{\sqrt{3}} \sum_i H_i = H
\end{equation}

$\theta^\alpha$ and $\bar{\tau}^{\dot{\alpha}}$ are two goldstinos. In order to have only $\theta^\alpha$ mode for $N = 1$ supersymmetry we put one instanton on O-plane.

\begin{equation}
\theta^\alpha = 1
\end{equation}

\begin{equation}
\bar{\tau}^{\dot{\alpha}} = 0
\end{equation}

These are O(1) instantons. There are additional zero modes if the cycles are not rigid. These are undesired zero modes. So, we will look only for O(1) rigid instantons.

\begin{equation}
I : \quad \Xi - D_a
\end{equation}

We have two bosonic twist fields $\prod_{i=1}^{2} \sigma_{1/2}^{i} S^\alpha V_{int}$ in complexified coordinates. In the NS sector we have integer moded fermionic modes and in R sector half-integer moded modes.

In NS sector:

\begin{equation}
V_{-1} = e^{-\phi} \prod_{i=1}^{2} \sigma_{1/2}^{i} S^\alpha V_{int}
\end{equation}

\begin{equation}
V_{-1} = e^{-\phi} \prod_{i=1}^{2} \sigma_{1/2}^{i} \bar{S}^{\dot{\alpha}} V_{int}
\end{equation}

\begin{equation}
[V_{-1}] = \frac{1}{2} + 2 \left( \frac{1}{2} \right)^2 + 2 \left( \frac{1}{2} \right)^2 + [V_{int}] = 1 + [V_{int}]
\end{equation}

We get massive modes. Bosonic massless modes are absent.
In R sector:

\[
V_{-1/2} = e^{-\frac{\phi}{2}} \prod_{i=1}^{2} \sigma_{1/2}^i V_{\text{int}}
\]  \hspace{1cm} (55)

\[
\theta_1, \theta_2 > 0, \quad \theta_3 < 0 \text{ then }
V_{\text{int}} = \prod_{i=1}^{2} \sigma_{\theta_i} e^{i(\theta_i - \frac{1}{2})H_i} \sigma_{H_{\theta_3}} e^{i(\frac{1}{2} + \theta_3)H_3}
\]  \hspace{1cm} (56)

\([V_{-\frac{1}{2}}] = 1 \text{ has massless zero modes. We denote by } \lambda_a \text{ the state } V_{-\frac{1}{2}}. \text{ It is charged under the brane, carries an index } a, \text{ and it is in the fundamental representation } [-1_E, \square].
\]

\[
\lambda_a = |\Xi \circ \pi_a| + |\Xi \circ \pi'_a|
\]  \hspace{1cm} (57)

Number of zero modes is equal to the number of intersections of instanton and brane.

\[
\Xi \circ \pi_a > 0 \quad [-1_E, \square] \]  \hspace{1cm} (58)

\[
\Xi \circ \pi_a < 0 \quad [1_E, \square] \]  \hspace{1cm} (59)

\[
\Xi \circ \pi'_a > 0 \quad [-1_E, \square] \]  \hspace{1cm} (60)

\[
\Xi \circ \pi'_a < 0 \quad [1_E, \square] \]  \hspace{1cm} (61)

The charge that zero modes carry is given by

\[
Q^a_\lambda = \mathcal{N}_a(\Xi \circ \pi_a - \Xi \circ \pi'_a)
\]  \hspace{1cm} (63)

\[
\int DX_E^{\mu} \int D\theta^\alpha \int \prod_{I,i} 2\lambda_i e^{-S_{cl} - S_{\text{int}}(\lambda, \theta^\alpha, \Phi^i)} = S_{\text{n.p.}}^{4D}(\phi^i, \psi_\alpha)
\]  \hspace{1cm} (64)

where \(\Phi = \phi + \theta^\alpha \psi_\alpha\).
Figure 11: Yukawa coupling $\phi\lambda\lambda$

$$S_{int} = X^{k}_{(Ii)(Jj)} \phi^k \lambda^i_I \lambda^j_J \theta^a \psi^a_k \lambda^i_I \lambda^j_J$$

each $\lambda^j_I$ appears exactly ones.

$$S^{4D}_{n.p.} = \int_{n.p.} DX^\mu \int D^2 \theta e^{-S_{cl}} \prod_{k, I_i, J_j} X^{k}_{I_i, J_j} \Phi^k$$

$$\nu_R = (-1_a, +1_b)$$

$\nu_R \nu_R$ coupling has deficit of charge $(-2_a, +2_b)$. We have to insure that

$$\Xi \circ \pi_a = 2$$

$$\Xi \circ \pi_b = -2$$

First equation means two copies of modes of charge $(-1_E, 1_a)$, and the second one two copies of modes of charge $(+1_E, -1_b)$. $\Xi \circ (SM) = 0$ and all their images of $(\pi'_a, \pi'_b)$.

Figure 12:

We choose $U(5)_a \times U(1)_b$. We want to introduce instanton that will create the exact number of zero modes which is 5. Single zero mode already carries charge 5.

$$\lambda_a = (+1_E, 1_a)$$

$$\lambda_b = (-1_E, 1_b)$$
\[ \Xi \circ \pi_a = -1 \]  
\[ \Xi \circ \pi_b = +1 \]  

Figure 13:

More general instantons

- Gauge instantons. There are more zero modes and more interactions among zero modes. There is a contribution of instantons that do not seat on the O-plane.

- \( U(1) \) instantons. \( U(1) \times U(1) \rightarrow O(1) \). There is a large number of zero modes.

- Instantons far away from O-plane, but intersecting with many zero modes. Also \( \bar{\tau} \) modes.

Mass terms are \((\theta_1 + \theta_2)\mu\bar{\mu}' + (-\theta_3)\mu'\bar{\mu}\). Susy term: \( -\theta_3(\mu\bar{\mu}' + \bar{\mu}\mu' + w'^\alpha\bar{w}_\alpha) \). We have also coupling to \( \bar{\tau} \): \( \tau_\alpha(w'^\alpha\mu + \bar{\mu}w^\alpha) \).