The scope of the landscape: supergravity and string vacua in 10D, 8D, 6D, and 4D

Part I: Supergravity and strings in 10D

TASI, June 2010
Boulder, Colorado

June 1, 2010

Washington (Wati) Taylor, MIT
Introduction

QFT: powerful framework ⇒ Standard model, many possible extensions
But: many possible QFT’s, few constraints

Question motivating these lectures:

Does gravity + UV consistency constrain low-E* physics (QFT)?
(* low-E = sub-Planck scale)

Study two aspects of global picture
1) Apparently consistent gravity + YM ($\mathcal{G}$)
2) Known string vacua ($\mathcal{V}$)

$\mathcal{G} \setminus \mathcal{V} =$ apparent “swampland” [Vafa]
Focus on supersymmetric theories. Why SUSY?

Phenomenological answers:

- Coupling constant unification
- Dark matter candidate
- Hierarchy problem

String theory answers:

- String theory needs SUSY at Planck scale
- Supersymmetry makes things simpler

Philosophy of these lectures: split by scale

\[
\begin{align*}
\Lambda & : \text{SUSY QFT} \\
\Lambda & : \text{SUGRA/strings}
\end{align*}
\]

If SUSY breaks at Planck scale:

UV physics very hard to understand
Plan of lectures:
Describe set of SUGRA + string theories with minimal SUSY in $D = 10, 8, 6, 4$

As we decrease dimensions:
- Theories richer, more structure
- Wider range of vacuum constructions
- Global description of landscape harder

10D: SUGRA, anomaly constraints, strings, branes
8D: Heterotic and F-theory compactifications
6D: Anomaly constraints, intersecting brane models, heterotic and F-theory cpt.
4D: IBM; IIB, IIA, NG flux vacua
I. Supergravity and string vacua in 10D

Supersymmetry:

- Symmetry relating bosons to fermions

- SUSY algebra: extends Poincare by fermionic generators
  \[
  \{ Q_\alpha, \bar{Q}_\beta \} = 2 \Gamma^\mu_{\alpha\beta} P_\mu
  \]
  basic action: \( \delta \phi \sim \bar{\epsilon} \psi, \quad \delta \psi \sim \Gamma^\mu \epsilon \partial_\mu \phi \)

- In some cases, multiple SUSY’s \(\rightarrow\) generators \(Q^A_\alpha, \quad A = 1, \ldots N\)
  \[
  \{ Q^A_\alpha, \bar{Q}^B_\beta \} = 2 \delta^{AB} P_\mu \Gamma^\mu_{\alpha\beta}
  \]

- Can extend by central charges \(\rightarrow\) identify topological charges
  \[
  \{ Q^A_\alpha, \bar{Q}^B_\beta \} = 2 \delta^{AB} P_\mu \Gamma^\mu_{\alpha\beta} + Z^{AB} \delta_{\alpha\beta}
  \]

- SUSY + gravity \(\Rightarrow\) local SUSY = supergravity

- Details of SUSY + spinors in various dimensions: \textbf{Polchinski App. B (v2)}
Supergravity in maximum dimension: $D = 11$

In $D > 11$, any representation of $\Gamma$’s (Clifford algebra) $\geq 64$-dimensional
$\Rightarrow$ massless particles of spin $> 2$ – no known (interacting) realizations.

Maximum $D$ for SUGRA: $D = 11$, $\mathcal{N} = 1$, 32 supercharges $Q_\alpha$

Fields:

\[
\begin{align*}
&g_{\mu\nu} \quad \text{graviton (metric)} \quad \frac{9\times10}{2} - 1 = 44 \text{ DOF} \\
&C_{\mu\nu\lambda} \quad \text{antisymmetric 3-form} \quad \frac{9\times8\times7}{6} = 84 \text{ DOF} \\
&\psi_{\mu\alpha} \quad \text{graviton (metric)} \quad 128 \text{ DOF}
\end{align*}
\]

Bosonic DOF = Fermionic DOF = 128

Action:

\[
S = \frac{1}{2\kappa_{11}^2} \left[ \int \sqrt{g} \left( R - \frac{1}{4} |F|^2 \right) - \frac{1}{6} \int C \wedge F \wedge F \right]
\]

$F^{(4)} = dC^{(3)}$

No gauge symmetries, matter. Global picture: $\mathcal{G}_{11} = \{M_{11}\}$ (= $\mathcal{V}_{11}$; M-theory)
10D supergravity

32 supercharges: IIA, IIB

IIA:

11D SUGRA

\[ \downarrow (S^1) \]

\[ \mathbb{R}^{1,9} \]

\[ g_{\mu\nu}^{(11)} \quad g_{\mu 11}^{(11)} \quad g_{11 11}^{(11)} \quad C_{\mu\nu\lambda}^{(11)} \quad C_{\mu\nu 11}^{(11)} \]

- In 10D: \( \Gamma^{11} = \prod_{\mu=0}^{9} \Gamma^\mu \); Can have 16, 16’ Majorana-Weyl chiral spinors.
- In IIA: two SUSY’s (\( \mathcal{N} = 2 \)), \( Q^1 \in 16, Q^2 \in 16' \)
- Bosonic fields \( g_{\mu\nu}, B_{\mu\nu}, \phi \) in \( \mathcal{N} = 1 \) multiplet
- \( A_\mu, C_{\mu\nu\lambda} \) “R-R fields” (name from string construction)

IIB:

- Two M-W SUSY’s with same chirality \( Q^1, Q^2 \in 16 \)
- Fields: \( g_{\mu\nu}, B_{\mu\nu}, \phi; \chi, \tilde{B}_{\mu\nu}, D^{+}_{\mu\nu\lambda\sigma} \)

Type IIA, IIB supergravity theories uniquely determined by SUSY
10D $\mathcal{N} = 1$ SUGRA: 16 supercharges $Q_\alpha \in 16$

SUSY multiplets:

\[
\begin{aligned}
S & \sim \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{g} \left[ e^{-2\phi} \left( R + 4\partial_\mu \phi \partial^\mu \phi - \frac{1}{2} |H|^2 \right) - \frac{1}{g_{YM}^2} |F|^2 \right] \\
H &= dB - \omega_Y \\
d\omega_Y &\sim \text{tr } F \wedge F
\end{aligned}
\]

From classical supergravity point of view:

\textbf{Appears that gauge group } G \textbf{ can be anything}

Does $\mathcal{G}_{10}$ contain infinitely many discrete components?

\textbf{No: quantum constraints from anomalies}
Anomalies

In 4D, chiral anomaly
\[ \partial_{\mu} j^5 \sim F \wedge F \] [see e.g. Peskin/Schroeder]

• Can be understood in terms of lack of invariance of measure in PI
\[ \int d\psi d\bar{\psi} \neq \int d\psi' d\bar{\psi}' \]

• Related to index theorem
• Gives breakdown of gauge invariance w/chiral fields
  anomalies in local symmetry ⇒ quantum inconsistency

In dimensions \( D = 4k + 2 \), Weyl representations \textit{self-conjugate}
⇒ particle + antiparticle have same chirality
⇒ gravitational (+ gauge, mixed) anomalies

10D anomalies: \( R^6, R^4 F^2, R^2 F^4, F^6 \rightarrow (D + 2)\)-form \( \hat{I} \)
Summary of 10D anomalies [Alvarez-Gaume/Witten]

Anomalies from $n$ spinors (8), gravitino (56), and (anti-)self-dual $D_{\mu \nu \lambda \sigma}^\pm$ (70)

\[
\begin{align*}
\hat{I}_8 &= -\frac{\text{Tr}(F^6)}{1440} + \frac{\text{Tr}(F^4)\text{tr}(R^2)}{2304} - \frac{\text{Tr}(F^2)\text{tr}(R^4)}{23040} - \frac{\text{Tr}(F^2)[\text{tr}(R^2)]^2}{18432} \\
&\quad + \frac{n \text{tr}(R^6)}{725760} - \frac{n \text{tr}(R^4)\text{tr}(R^2)}{552960} + \frac{n [\text{tr}(R^2)]^3}{1327104} \\
\hat{I}_{56} &= -495 \frac{\text{tr}(R^6)}{725760} - 225 \frac{\text{tr}(R^4)\text{tr}(R^2)}{552960} - 63 \frac{[\text{tr}(R^2)]^3}{1327104} \\
\hat{I}_{70} &= 992 \frac{\text{tr}(R^6)}{725760} + 448 \frac{\text{tr}(R^4)\text{tr}(R^2)}{552960} + 128 \frac{[\text{tr}(R^2)]^3}{1327104}
\end{align*}
\]

Type IIB: 2 $\zeta_\alpha$’s (8’, $n = -2$), 2 $\psi_{\mu \alpha}$ (56), 1 $D_{\mu \nu \lambda \sigma}^+$ (70), $F = 0$

\[-2\hat{I}_8 (F \to 0, n \to 1) + 2\hat{I}_{56} + \hat{I}_{70} = 0!!\]

$\mathcal{N} = 1$: no $\hat{I}_{70} \to$ no cancellation?
Green-Schwarz anomaly cancellation

Problem: $\mathcal{N} = 1$ SUGRA + YM group $G$ has hexagon anomalies $\sim (R, F)^6$.

Solution: careful treatment of $B$ couplings, new $B$ couplings at higher order

$$S_B \sim (dB - \omega)^2 + B \wedge d\tilde{\omega}$$

$$d\omega = c \text{ tr} F^2 + c' \text{ Tr} R^2 = Y_4(F, R)$$

$$d\tilde{\omega} = X_8(F, R)$$

Variation of $\delta B = c \text{ Tr}(\Lambda F) + c' \text{ tr}(\Theta R)$ where $\delta A = d\Lambda$, $\delta \omega_1 = d\Theta$$$

$\delta(B \wedge X_8) \neq 0$ can cancel anomalous variation of $\Pi$

Can cancel anomaly if $\hat{I}_{12} \sim Y_4X_8$ factorizes

Tree diagrams:

$$Y_4(F, R)X_8(F, R)$$
Anomaly cancellation for $\mathcal{N} = 1$ SUGRA, gauge group $G$

Fields: gravitino (56), neutral fermion (8'), $n(= \dim G)$ gauginos (8)

Want

$$\hat{I}_{12} = (n - 496) \frac{\text{tr}(R^6)}{725760} - \frac{\text{Tr}(F^6)}{1440} + \cdots$$

$$= Y_4(F, R)X_8(F, R)$$

For factorization: $R^6$ term must cancel $\Rightarrow n = 496$

$F^6$ cancellation + factorized form only if

$$\text{Tr}(F^6) = \frac{1}{48} \text{Tr}(F^2)\text{Tr}(F^4) - \frac{1}{14400} [\text{Tr}(F^2)]^3$$

Only satisfied for 4 groups: $SO(32), E_8 \times E_8, U(1)^{496}, E_8 \times U(1)^{248}$

So, including IIA, IIB only [6 candidates for UV-consistent SUGRA] in 10D.
UV completions of 11D + 10D SUGRA

Rely on extended objects

Just as similarly

pointlike particle couples to $A_\mu$

$p$-brane couples to $(p + 1)$-form $A_{\mu_1 \cdots \mu_{p+1}}$

<table>
<thead>
<tr>
<th>Theory</th>
<th>Field</th>
<th>Brane</th>
</tr>
</thead>
<tbody>
<tr>
<td>11D</td>
<td>$C_{\mu \nu \lambda}$</td>
<td>M2-brane (+ dual M5)</td>
</tr>
<tr>
<td>$\mathcal{N} = 1, 2$ 10D</td>
<td>$B_{\mu \nu}$</td>
<td>(F) string (+ NS5-brane)</td>
</tr>
<tr>
<td>IIA</td>
<td>$A_\mu, C_{\mu \nu \lambda}, \tilde{B}<em>{\mu \nu}, D</em>{\mu \nu \lambda \sigma}^+$</td>
<td>(D)-branes</td>
</tr>
<tr>
<td>IIB</td>
<td>[RR-fields]</td>
<td></td>
</tr>
</tbody>
</table>
Brane democracy
Quantize any brane ⇒ Quantum gravity (but usually hard except in limits)

11D supergravity (M-theory):
M2-brane: In light cone ⇒ M(atrix) theory (also from D0)

10D supergravity:

$D_p$-branes: Near horizon limit ⇒ AdS/CFT
- $A_\mu, X_\mu$ fields on D-brane → SYM

Strings (F1-branes): perturbative string theory
- Best understood in $g, B, \phi$ background; RR bg tricky [see Berkovits]
- D-branes: loci for open string endpoints
- Strings between branes → nonabelian $G \rightarrow SU(N)$
- $\mathcal{N} = 2$: IIA, IIB; $\mathcal{N} = 1$: heterotic: $E_8 \times E_8, SO(32)/\mathbb{Z}_2 = \text{type I}$
Summary of global situation in 10D

6 distinct theories w/o known inconsistencies; 4 realized in string theory
2 with no known string realization–apparent swampland

Remaining challenge:
Find string realization or UV inconsistency
for $U(1)^{496}$ and $E_8 \times U(1)^{248}$ theories

[Update: inconsistency may be proven, stay tuned [Adams/de Wolfe/WT]]

Henceforth, focus on nonabelian part of gauge groups