Mass and Spin Measurement Techniques
(for the Large Hadron Collider)

Based on “A review of Mass Measurement Techniques proposed for the Large Hadron Collider”, Barr and Lester, arXiv:1004.2732

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A Review of the Mass Measurement Techniques proposed for the
Large Hadron Collider

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We review the methods which have been proposed for measuring masses of new particles at the Large Hadron Collider paying particular attention to the kinematical techniques suitable for extracting mass information when invisible particles are expected.
Scope and disclaimers

– am not interested in fully visible final states as standard mass reconstruction techniques apply
– will only consider new particles of unknown mass decaying to invisible particles of unknown mass (and other visible particles)
– selection bias – more emphasis on things I’ve worked with
  • Transverse masses, MT2, kinks, kinematic methods.
  • (Not Matrix Element / likelihood methods / loops)
– not shameless promotion – focus on faults!

Sneak peek at conclusions

• Don’t trust experimental collaborations. They are probably doing the wrong thing.

• If you can’t understand why the experimental paper says the experiment did, it might be because they don’t know either (sphericity)
Recall there are some problems

Aim was to fix some of these problems with the Standard Model

- Fine-tuning / “hierarchy problem” (technical) – Why are particles light?
- Does not explain Dark Matter
- No gauge coupling unification

What are common features of “solutions” to these problems?

- Big increase in particle content
- Longish decay chains
- Missing massive particles
- Large jet/lepton/photon multiplicity
The game...

40 M / second over 10 years

+ more terms...?

At some point, 5000 people will shout:

“We’ve found a ... [long pause] ... SOMETHING!”

A large collider of hadrons ...
... not a collider of large hadrons
How hard is it to identify what was found?

Want to emphasise what is visible at the LHC

- Distinguish the following from each other
  - Hadronic Jets,
    - B-jets (sometimes)
  - Electrons, Positrons, Muons, Anti-Muons
    - Tau leptons (sometimes)
  - Photons

- Measure Directions and Momenta of the above.

- Infer total transverse momentum of invisible particles. (e.g., neutrinos)

What do we NOT measure?
What might events look like?

This is the high energy physics of the 21st Century!

What events really look like scares me!

An example of an event where a higgs boson decayed to a pair of b-quarks/soft gluon radiation?
Supersymmetry as Lingua Franca

Some possibilities:

- **Supersymmetry**
  - Minimal
  - Non-minimal
  - R-parity violating or conserving

- **Extra Dimensional Models**
  - Large (SM trapped on brane)
  - Universal (SM everywhere)
  - With/without small black holes

- “Littlest” Higgs?
- ...

We will look mainly at supersymmetry (SUSY)

Supersymmetry!

**CAUTION!**

- It may exist
- It may not
- First look for deviations from Standard Model!

Gamble:

**IF DEVIATIONS ARE SEEN:**

- Old techniques won’t work
- New physics not simple
- Can new techniques in SUSY but can apply them elsewhere.
You know what Supersymmetry is:

**Matter**
- Electron
- Higgs
- Selectron
- Higgsino

**Antimatter**
- Anti-Electron
- Higgs
- Anti-Selectron
- Anti-Higgsino

Reverse the charges, retain the spins.

Retain the charges, reverse the spins. (exchange boson with fermions).

**Supersymmetric Matter**

**SUSY particle content**

<table>
<thead>
<tr>
<th>Spin-1/2</th>
<th>SM</th>
<th>SUSY</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>quarks (L&amp;R)</td>
<td>squarks (L&amp;R)</td>
</tr>
<tr>
<td></td>
<td>leptons (L&amp;R)</td>
<td>sleptons (L&amp;R)</td>
</tr>
<tr>
<td></td>
<td>neutrinos (L&amp;R)</td>
<td>sneutrinos (L&amp;R)</td>
</tr>
<tr>
<td>Spin-1</td>
<td>[\gamma, Z^0, W^\pm]</td>
<td>Bino, Wino</td>
</tr>
<tr>
<td></td>
<td>gluon</td>
<td>[\tilde{\chi}^0_1, \tilde{\chi}^0_2, \tilde{\chi}^0_3, \tilde{\chi}^0_4]</td>
</tr>
<tr>
<td>Spin-0</td>
<td>[\tilde{h}_0, \tilde{H}_0]</td>
<td>[\tilde{\tilde{\chi}}^0_1, \tilde{\tilde{\chi}}^0_2]</td>
</tr>
<tr>
<td></td>
<td>[\tilde{H}_0]</td>
<td>[\tilde{\tilde{\chi}}^\pm_1, \tilde{\tilde{\chi}}^\pm_2]</td>
</tr>
</tbody>
</table>

All that concerns us is that SUSY has a menu of particles.
Even in SUSY many possibilities

RPV
(Baryon number violating)

RPV
(Lepton number violating)

RPC

Do we care about masses?
- Common Parameter in the Lagrangian
- Expedites discovery – optimal selection
- Interpretation
  (SUSY breaking mechanism, Geometry of Extra Dimensions)
- Prediction of new things
  Mass of W,Z \(\rightarrow\) indirect top quark mass “measurement”
“mass measurement methods”

… short for …

“parameter estimation and discovery techniques”
More Realistic Hadron Collider

Types of Technique

Few assumptions
- Missing transverse momentum
- M_{eff}, H_T
- s Hat Min
- M_T
- M_{TGEN}
- M_{T2} / M_{CT}
- M_{T2} (with “kinks”)
- M_{T2} / M_{CT} (parallel / perp)
- M_{T2} / M_{CT} ("sub-system")
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Many assumptions

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Types of Technique

Vague conclusions
- Missing transverse momentum
- \(M_{\text{eff}}, H_T\)
- \(s \text{ Hat Min}\)
- \(M_T\)
- \(M_{\text{TGEN}}\)
- \(M_{T2} / M_{CT}\)
- \(M_{T2}\) (with “kinks”)
- \(M_{T2} / M_{CT}\) (parallel / perp)
- \(M_{T2} / M_{CT}\) (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Specific conclusions

Robust
- Missing transverse momentum
- \(M_{\text{eff}}, H_T\)
- \(s \text{ Hat Min}\)
- \(M_T\)
- \(M_{\text{TGEN}}\)
- \(M_{T2} / M_{CT}\)
- \(M_{T2}\) (with “kinks”)
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- \(M_{T2} / M_{CT}\) (“sub-system”)
- “Polynomial” constraints
- Multi-event polynomial constraints
- Whole dataset variables
- Cross section
- Max Likelihood / Matrix Element

Fragile
Interpretation: the balance of benefits

- Few assumptions
- Vague conclusions
- Robust

- Many assumptions
- Specific conclusions
- Fragile

Topology / hypothesis

Full index in arXiv:1004.2732
Topology / hypothesis

Must impose some interpretation

Design the variable to suit the interpretation

Full index in arXiv:1004.2732

Lectures are roughly ordered from simple to complicated ...

(more details in arXiv:1004.2732)
... and from **few** events required, to **many** events required ....

Good vs poor variables

“Goodness” can be formalised: cartoons just for demonstration
Few assumptions, Vague Conclusions.

Anything with sensitivity to mass scales.
Missing transverse momentum

\[ \vec{p}_T^{\text{miss}} = -\sum_i \vec{p}_T^{i\text{th} \text{visible}} \]

Events have missing energy too, and it’s not missing momentum

Total 4-momentum of invisibles.

Missing energy could be big, even if missing transverse momentum is small.

Can’t measure E or pz.
Rant about missing transverse momentum

- eTmiss – aaargh
- MET – AAAARGH
- missing energy – AAAAAARRRGGH

- Blame LEP?
- Calorimeter apologists?

- alphaT

Main EASY signatures are:

- Lots of missing pt
- Lots of leptons
- Lots of jets

Simply counting events
Perhaps

simple = best?

The End

Can attempt to spot susy by counting “strange” events …

… but can we say anything concrete about a mass scale?

Next example still low-tech …
Effective mass

What you histogram: \[ M_{\text{eff}} = p_T^{\text{missing}} + \sum_i |p_T^{\text{jet}_i}| \]

You look for position of this peak and call it MeffPeak

Call it Meff and Mest too (just to confuse people!)

What might Meff peak position correlate with?

Define SUSY scale:

\[ M_{\text{eff}}^{\text{susy}} = \left( M_{\text{susy}} - \frac{M^2_{\chi}}{M_{\text{susy}}} \right), \text{ with } M_{\text{susy}} \equiv \frac{\sum_i M_i \sigma_i}{\sum_i \sigma_i} \]
Observable $M_{\text{effPeak}}$ sometimes correlates with property of model $M_{\text{eff}}$ defined by

$$M_{\text{effSusy}} = \left( M_{\text{Susy}} - \frac{M_{\chi}^2}{M_{\text{Susy}}} \right)$$

but correlation is model dependent.
M_Hotpants ..

- Can encourage tendency to
- Create your variable, then see what might be able to measure. Oops.

Effective mass

\[ M_{\text{eff}} = p_{\text{missing}}^T + \sum |p_T| \]

You look for position of this peak and call

"It is neither a mass, nor effective" - KM

Call it Meff too (just to confuse people!)
Meff is not alone …

Murky underworld of badly formed twins known variously as HT … the less said the better

\[ H_T = E_T(2) + E_T(3) + E_T(4) + |\vec{p}_T| \]

\[ E_T = E \sin \theta \]

See arXiv:1105.2977 for why sinTheta brings on nightmares.

(There are no standard definitions of \( H_T \) authors differ in how many jets are used, whether PT miss should be added etc.)

All have some sensitivity to the overall mass scales involved, but interpretation requires a model and more assumptions.

Why are we adding transverse momenta?

• Why not multiply? (or add logs)?

\[ M_{\text{happy}} = \left( \prod_{i=1}^{n} p_i^T \right)^{\frac{1}{n}} \]

• Serious proposal to use Meff\(^2\)-(u_T)\(^2\) in arXiv:1105.2977

• Why are the signs the same? Why equal weights? Silly?

• How many years would it take ATLAS/CMS to discover the invariant mass for \( Z \rightarrow a \ b \)?

\[ M^2 = \left( \sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2} \right)^2 \]

\[ - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2 \]
Latest ATLAS 0-lepton, jets, missing transverse momentum data.
The highest Meff in any (supposedly “clean”) ATLAS event is 1548 GeV – calculated from four jets with pts:
- 636 GeV
- 189 GeV
- 96 GeV
- 81 GeV
- 547 GeV of missing transverse momentum.
Don’t confuse simplicity with complexity … can layer add many layers of interpretation.
Measure top quark mass from mean lepton PT only!

A new measurement of the top quark mass at 1.8 fb$^{-1}$ integrated luminosity, using leptons' $P_T$ in the dilepton channel is presented. A top quark mass of $m_{\text{top}}=156\pm20_{\text{stat}}\pm4.6_{\text{syst}}$ GeV$/c^2$ is obtained with the Likelihood method and of $149\pm21_{\text{stat}}\pm5_{\text{syst}}$ GeV$/c^2$ is obtained with the Straight Line method.

Top quark production tevatron

- Hadron 1
- Hadron 2
- Remnant 1
- Remnant 2
- Other stuff
- neutrino
- W
- lepton
- jet
Frightening y-axis!

Simulated top quark mass

Result

\[ m_{\text{top}} = 156 \pm 20_{\text{(stat)}} \pm 4.6_{\text{(syst)}} \text{GeV} \]

Moral

• You can monte-carlo anything.
  – example h->tau tau

• But do you trust it? Is it the best you can do?
More assumptions
Less Vague Conclusions

non-hotpants

Topology / hypothesis

Must impose some interpretation

Design the variable to suit the interpretation

Full index in arXiv:1004.2732
All visible
\[ Z^0 \rightarrow e^+ e^- \]

On-shell, perfect measurement

Counts

On-shell, perfect measurement

\[ f^2 = Z^0 e^+ e^- (a+b) \mu (a+b) \mu \]

\[ M^2 = \left( \frac{m_a^2 + m_b^2}{m_a + m_b} + \sqrt{m_a^2 + m_b^2 + b_x^2 + b_y^2 + b_z^2} \right)^2 - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2 \]

SPS – the Z boson Mass

Finite width
Detector resolution
Broaden peak

UA1 CERN 1989
Dealing with incomplete information

Observe: $P_e$ (four components)

Unobserved: $P_\nu$ (does not interact)

Cannot reconstruct $(P_\nu + P_e)^2$

Unobserved, but not unconstrained...

**Image:**
- **ATLAS Experiment**
- **ELECTRON**
- **Missing momentum**
Historical solution:
(full!) W transverse mass

\[ m_T^2 = m_e^2 + m_\nu^2 + 2(e_e e_\nu - \mathbf{p}_T e \cdot \mathbf{p}_T \nu) \]

\[ e_e = \sqrt{m_e^2 + p_T^2} \]
\[ e_\nu = \sqrt{m_\nu^2 + p_T^2} \]

!! NOT THIS !!

\[ m_T = \sqrt{2|\vec{P}_{T\nu}|(1 - \cos \theta)} \]

!! This is NOT the transverse mass !!

W transverse mass: nice properties

- In every event \( m_T < m_W \) if the W is on shell
- There are events in which \( m_T \) can saturate the bound on \( m_W \).

motivate \( m_T \) in W discovery and mass measurements.

But where did these properties come from?
Re-examine invariant mass: \( M \rightarrow a\ b \)

\[
M^2 = \left( \sqrt{m_a^2 + a_x^2 + a_y^2 + a_z^2} + \sqrt{m_b^2 + b_x^2 + b_y^2 + b_z^2} \right)^2
- (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2
\]

\[
= (E_a + E_b)^2 - (a_x + b_x)^2 - (a_y + b_y)^2 - (a_z + b_z)^2
\]

\[
= m_a^2 + m_b^2 + 2(E_aE_b - a_xb_x - a_yb_y - a_zb_z)
\]

\[
= m_a^2 + m_b^2 + 2(e_a e_b \cosh(\Delta \eta) - a_x b_x - a_y b_y)
\]

where \( e_a = \sqrt{m_a^2 + a_x^2 + a_y^2} \) and \( e_b = \sqrt{m_b^2 + a_x^2 + a_y^2} \) and

\[
\eta_a = \frac{1}{2} \ln \left( \frac{E_a + a_z}{E_a - a_z} \right) \quad \eta_b = \frac{1}{2} \ln \left( \frac{E_b + b_z}{E_b - b_z} \right) \quad \Delta \eta = \eta_a - \eta_b
\]

Comparing invariant and transverse masses:

\[
M^2 = m_a^2 + m_b^2 + 2(e_a e_b \cosh(\Delta \eta) - a_x b_x - a_y b_y)
\]

\[
M_T^2 = m_a^2 + m_b^2 + 2(e_a e_b - a_x b_x - a_y b_y)
\]

Since \( \cosh(\Delta \eta) \geq 1 \) have \( M_T \leq M \)

with equality when \( \Delta \eta = 0 \).

(Not same as throwing away z information!)

But have bound, and bound can be saturated.

Note that at this point we are assuming we know \( m_b \).
W boson mass measurement

Plot $m_T$ for each event

Each new event gives a new lower bound on $m_W$

If bound is saturated (as it is in this example) the endpoint is $m_W$

In the data....

Bound at $m_W$ smeared by resolution and finite width effects

Monte Carlo modelling

Alternative way of approaching the problem

Set out **INTENDING** to construct best lower bound
on \((P_e + P_\nu)^2\)
given the constraints

**Constraints in this instance:**
0 = \((P_\nu)^2\) \[massless neutrino]\n0 = \(\Sigma p_T = u_T + p_T(e) + p_T(\nu)\) \[momentum conservation in transverse plane]\n
**Exercises**

\(M \rightarrow a\ b\)

1. Prove that
\[M^2 = m_a^2 + m_b^2 + 2(e_a e_b \cosh(\Delta \eta) - a_x b_x - a_y b_y)\]

2. We have shown that \(M_T\) (at fixed and correct \(m_b\)) is an observable that is bounded above by \(M\) for unsmeared signal events \(M \rightarrow a\ b\). Go further than this. Prove that it is **the greatest possible** lower bound for the mass of the parent.

3. It is trivial to demonstrate that \(M_T\) is invariant under longitudinal boosts. Is it invariant under transverse parental boosts? What about the kinematic endpoint of the \(M_T\) distribution?
Suggests general prescription...

(1) Propose a decay **topology**
(2) Write down your the **Lorentz Invariant** of choice
(3) Write down the **constraints**
(4) **Calculate** the bound (algebraically/numerically/mix)

\[
\begin{align*}
(1) & \quad P \quad \sum_{i=1}^{N_\nu} \bar{q}_{iT} = \bar{p}_T \equiv -\bar{u}_T - \sum_{i=1}^{N_\nu} \bar{p}_{iT} \\
(2) & \quad M_\alpha \equiv \sqrt{g_{\mu\nu} (P_\alpha + Q_\alpha)^\mu (P_\alpha + Q_\alpha)^\nu} \\
(3) & \quad M_{1T}^2 = \left(\sqrt{M_P^2 + \vec{p}_T^2} + \sqrt{M_{\text{slash}}^2 + \vec{q}_{T\text{miss}}^2}\right)^2 - u_T^2 \\
(4) & \quad M_{\text{slash}} = \sum_i \tilde{M}_i
\end{align*}
\]

**Single parent ... multiple daughters**

- many visible
- many invisible

Bound depends on **GUESS** masses of **all** invisible daughters
- Most conservative: set to zero
- [more later]
Almost exactly same as transverse mass – one small generalization

\[
M_{1T}^2 = \left(\sqrt{M_P^2 + \vec{p}_T^2} + \sqrt{M_{\text{slash}}^2 + \vec{q}_{T\text{miss}}^2}\right)^2 - u_T^2
\]

\[
M_T^2 = \left(\sqrt{M_P^2 + \vec{p}_T^2} + \sqrt{M_Q^2 + \vec{q}_{T\text{miss}}^2}\right)^2 - u_T^2
\]

The “invisible mass” has become a parameter …. rather than the actual visible mass.

We will come back to this many times.

Suggests we should think about non-physical parameters a bit more ….

Applications of \(M_{1T}\)?
Higgs $\rightarrow$ WW$^*$ $\rightarrow$ lvlv

Written up in http://arxiv.org/abs/1106.2322

Why are endpoints often more robust than shapes?

FIG. 1: Signal-only distributions of $m_{WW}^{\text{true}}$ (top) and $m_{WW}^{\text{true}}$ (bottom) for various values of $m_h$ (in GeV). No cuts on $\Delta \phi_{Q^1}$ and $p_{TWW}^{\text{min}}$ have been applied.

Previous variable (not a bound)

Proper bound var $MT\text{True} = MT$
Application to Higgs→WW→lνlν

Bound worsens as information is removed...

Blue = BEST lower bound
Red = unknown z components of electrons (worse)
Green = worse again

Simulation: 200 GeV
Higgs boson

Against the 2010 LHC data...

ATLAS-CONF-2011-005

170 GeV
Higgs boson

Big improvement in LHC Higgs Search
Other applications of $M_{1T}$?
\( \sqrt{\hat{S}}_{\text{min}} \) is fully inclusive \( M_{1T} \) (i.e. \( u_T=0 \))

\( \sqrt{\hat{S}}_{\text{min}} \) seeks to bound the invariant mass of the interesting part of the collision

\[
\frac{1}{2} \sqrt{\hat{S}}_{\text{min}} = (E^2 - P_Z^2)^{\frac{1}{2}} + (P_T + M_{\text{invis}})^{\frac{1}{2}}
\]


Without ISR / MPI

From arXiv:0812.1042
With ISR & MPI etc

![Graph showing the effect of ISR and MPI contamination](image)

From arXiv:0812.1042

Effect of ISR and MPI contamination

![Graph showing the effect of ISR and MPI contamination](image)

Though dependence on ISR is large, it is calculable and may offer a good test of our understanding. See arXiv:0903.2013 and 1006.0653
Moral

• Remember our variables are always limited by what we feed them
  – (garbage in garbage out)
• May need alter variable in light of pathologies
  – Try to locate the subsystem that lacks ISR/FSR with rapidity and pt cuts.
  – This takes away $u_T=0$ requirement, and gets us back to $M_{1T}$ (a.k.a. “subsystem root s hat min”)

An example with additional (internal) constrains …
Example with additional internal constraints

\[ Q_1^\mu Q_1\mu = 0, \]
\[ Q_2^\mu Q_2\mu = 0, \]
\[ (Q_1^\mu + P_1^\mu)(Q_1\mu + P_1\mu) = m_1^2, \]
\[ (Q_2^\mu + P_2^\mu)(Q_2\mu + P_2\mu) = m_2^2, \]
\[ \vec{q}_{1T} + \vec{q}_{2T} = \vec{p}_T. \]

Written up in
http://arxiv.org/abs/1106.2322

Result

Parent mas bound (no intermediate constraint) = M1T
Including the intermediate constraint (BEST)
Not a bound (existing var)

Just the visible (existing var)

Dramatic difference to Higgs observability?

http://arxiv.org/abs/1106.2322
But what if we don’t know the masses of the invisible particles?

A popular new-physics scenario

Proton 1

Remnant 1

Upstream Momentum

Visible

Invisible

Proton 2

Remnant 2

Visible
Example:

We have two copies of this:

One copy could be just as relevant!
Have to go back to the (full) transverse mass again!

**Problem:** Bound **not** generally **saturated** if daughter masses unknown
Free to try **different guesses** of invisible particle masses

![Diagram](image)

WANT bound on $M_A$

BUT $M_B$ unknown...

Bound on $M_A$ depends on hypothesis of $M_B$

---

In next few slides:

$\chi = $ Guess (i.e. hypothesis) for mass of the invisible daughter

![Diagram](image)
Schematically, all we have guaranteed so far is the picture below:

- Since $\chi$ can now be "wrong", some of the properties of the transverse mass can "break":
  - $m_T(\chi)$ max is no longer invariant under transverse boosts! (except when $\chi=m_B$)
  - $m_T(\chi)<m_A$ may no longer hold! (however we always retain: $m_T(m_B) < m_A$)

Actual dependence on invisible mass guess $\chi$ more like this:
In fact, we get this very nice result:

The “full” transverse mass curve is the boundary of the region of (mother, daughter) masses consistent with the observed event!

Minimal Kinematic Constraints and m(T2), Hsin-Chia Cheng and Zhenyu Han (UCD) e-Print: arXiv:0810.5178 [hep-ph] and “Transverse masses and kinematic constraints, from the Boundary to the Crease” arXiv:0908.3779.

Exercise

• (4) Prove the happy-face/sad-face statement made on the previous slide.
• [Note: not same as exercise (2). There mass of invisible was fixed at true value. Here it is not.]
Event 1 of 8

$$m_T(\chi)$$

$$m_A$$

$$m_B$$

Event 2 of 8

$$m_T(\chi)$$

$$m_A$$

$$m_B$$
Event 3 of 8

$$m_T(\chi)$$

Event 4 of 8

$$m_T(\chi)$$
Overlay all 8 events

$\mathbf{m_T(\chi)}$

$\mathbf{m_A}$

$\mathbf{m_B}$

__CASE 2__

Overlay many events

$\mathbf{m_T(\chi)}$

$\mathbf{m_A}$

Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

CASE 2
Here is a transverse mass “KINK”

$\chi$

Alternatively, look at $M_T$ distributions for a variety of values of $\chi$.

Each curve has a different value of $\chi$

Where is the kink now?
What causes the kink?

• Two entirely independent things can cause the kink:
  – (1) Variability in the “visible mass”
  – (2) Recoil of the “interesting things” against Upstream Transverse Momentum

• Which is the dominant cause depends on the particular situation … let us look at each separately:

Kink cause 1: Variability in visible mass

• $m_{\text{vis}}$ can change from event to event
• Gradient of $m_T(\chi)$ curve depends on $m_{\text{vis}}$
• Curves with low $m_{\text{vis}}$ tend to be “flatter”
Kink cause 1: Variability in visible mass

• \( m_{\text{vis}} \) can change from event to event
• Gradient of \( m_T(\chi) \) curve depends on \( m_{\text{vis}} \)
• Curves with high \( m_{\text{vis}} \) tend to be “steeper”

Exercise: \( M \rightarrow (a_1a_2)b \)

For the three body decay \( M \rightarrow (a_1a_2)b \) where \( a_1 \) and \( a_2 \) are visibles of known masses, while the \( b \) is invisible.

• (5) Satisfy yourself that, at the true value of the invisible mass, events can have \( M_T \) values that saturate the bound (i.e. have \( M=M_T \)) regardless of the invariant mass “\( m_{\text{vis}} \)” of the \( a_1a_2 \) system.

• (6) Sketch a proof of the statements made in the last two slides – in some limit if necessary.
Kink cause 2: Recoil against Upstream Momentum

- UTM can change from event to event
- Gradient of $m_T(\chi)$ curve depends on UTM
- Curves with UTM parallel to visible momenta tend to be "flatter"
Kink cause 2: Recoil against UTM

• UTM can change from event to event
• Gradient of $m_T(\chi)$ curve depends on UTM
• Curves with UTM opposite to visible momenta tend to be “steeper”

Exercise

• (7) Sketch a proof of the statements of the last two slides (if necessary, only for special cases of your choice)
MT works for:

What do we do in events with a pair of decays?
MT2 : the stransverse mass

For a pair of decays

one can generalize $m_T$ to $m_{T2}$
("Transverse" mass to "Stransverse" mass)

$$m_{T2}(\chi) = \min_{\text{splittings}} \left( \max[m_T(\chi; \text{side 1}), m_T(\chi; \text{side 2})] \right)$$


As before: look for maximal lower bound for $M_1$
(see diagram below) subject to desired pair-production constraints.

Note, other approaches: MCT, Rogan, Razor, etc.

CONSTRANTS

$M_1 = M_2$

Momentum conservation in transverse plane
For each partition of missing $p_T$

Bound for this system: $m_T^{(a)}$

Bound for this system: $m_T^{(b)}$

Constructing the bound

$$m_{T2}(v_1, v_2, \not p_T, m_i^{(1)}, m_i^{(2)}) \equiv \min_{\sum q_r = p_T} \left\{ \max \left( m_T^{(1)}, m_T^{(2)} \right) \right\}$$

The most conservative partition consistent with the constraint

Take the better of the two lower bounds

The Cambridge “Stransverse Mass”

[Received six comments about “mis-spelling” of transverse in ATLAS editorial board!]
Properties of the $m_{T2}$ function

1. Identical pair decays
   $m_\gamma < m_{T2} < m_0$

2. Non-identical pair decays
   $m_\gamma < m_{T2} < \max(m_0,m_0')$

3. Small missing momentum
   $m_{T2} \rightarrow m_\gamma$ as $p_{T\text{miss}} \rightarrow 0$

4. Small jet momentum
   $m_{T2} \rightarrow m_\gamma$ as $p_{T\text{jet}} \rightarrow 0$

5. Jet $\parallel$ to missing
   $m_{T2} \rightarrow m_\gamma$ for $p_{T\text{miss}} \parallel p_{T\text{jet}}$

6. $m_{T2} \rightarrow m_\gamma$ for
   $p_{T\text{miss}} = \Sigma_i \alpha_i p_{T\text{jet}(i)}$ for $\alpha_i > 0$

7. 1-6 also true for composite systems

Graphically:

- 3-jet
- $Z \rightarrow \nu\bar{\nu} + \text{jet}$
- QCD plus mismeasured jet
- Detector effects

All these have $m_{T2}$ either $< m_{\text{top}}$ or $\rightarrow m_\gamma$

$m_{T2}$ adopts small values for a variety of interesting configurations

AJB and Gwenlan
arXiv:0907.2713
Example proof

**Lemma 4** When $p_T = 0$ and $m^{(1,2)}_i = 0$ then $m_{T2} = m_{<}$.

**Proof** For $p_T = 0$ there exists a trivial partition of the missing momentum with $q^{(1)}_T = q^{(2)}_T = 0$. For that partition, $m^{(1)}_T = m^{(1)}_{<}$ and $m^{(2)}_T = m^{(2)}_{<}$.

$m_{T2}(v_1, v_2, p_T; m^{(1)}, m^{(2)}) = \min_{q_T = p_T} \{ \max(m^{(1)}_T, m^{(2)}_T) \}$

- So small $p_T^{\text{miss}} \Rightarrow$ small $m_{T2}$
- Do we need a separate $p_T^{\text{miss}}$ cut? (no…)

NB the requirement that $m=0$ is on the input mass parameter not the true LSP mass

<table>
<thead>
<tr>
<th>Process</th>
<th>$m_{T2}(v_1, v_2, p_T, 0, 0)$</th>
<th>Comments</th>
</tr>
</thead>
<tbody>
<tr>
<td>QCD di-jet \rightarrow hadrons</td>
<td>$= \max m_j$ by Lemma 14</td>
<td>fully hadronic decays</td>
</tr>
<tr>
<td>QCD multi jets \rightarrow hadrons</td>
<td>$= \max m_j$ by Lemma 4</td>
<td>any leptonic decays</td>
</tr>
<tr>
<td>$t\bar{t}$ production</td>
<td>$= \max m_j$ by Lemma 4</td>
<td>fully hadronic decays</td>
</tr>
<tr>
<td>Single top / $tW$</td>
<td>$= \max m_j$ by Lemma 4</td>
<td>any leptonic decays</td>
</tr>
<tr>
<td>Multi jets: “fake” $p_T$</td>
<td>$= \max m_j$ by Lemma 5</td>
<td>single mismeasured jet</td>
</tr>
<tr>
<td>Multi jets: “real” $p_T$</td>
<td>$= \max m_j$ by Lemma 5</td>
<td>two mismeasured jets</td>
</tr>
<tr>
<td>$Z \rightarrow \nu\bar{\nu}$</td>
<td>$= 0$ by Lemma 3</td>
<td>single jet with leptonic $b$ decay</td>
</tr>
<tr>
<td>$Z \rightarrow \nu\bar{\nu} j$</td>
<td>$= m_j$ by Lemma 3</td>
<td>two jets with leptonic $b$ decays</td>
</tr>
<tr>
<td>$W \rightarrow \ell\nu$</td>
<td>$= m_\ell$ by Lemma 3</td>
<td>one ISR jet</td>
</tr>
<tr>
<td>$W j \rightarrow \ell\nu j$</td>
<td>$\leq m_W$ by Lemma 2</td>
<td>one ISR jet</td>
</tr>
<tr>
<td>$WW \rightarrow \ell\ell\nu$</td>
<td>$\leq m_W$ by Lemma 1</td>
<td>also $m_j$ for one ISR jet</td>
</tr>
<tr>
<td>$ZZ \rightarrow \nu\nu\nu$</td>
<td>$= 0$ by Lemma 3</td>
<td>i.e. can take large values</td>
</tr>
<tr>
<td>$LQ \rightarrow q\bar{q}\nu\bar{\nu}$</td>
<td>$\leq m_{LQ}$</td>
<td></td>
</tr>
<tr>
<td>$q \bar{q} \rightarrow q\chi_1^0 \bar{q}\chi_1^0$</td>
<td>$\leq m_{\tilde{q}}$</td>
<td></td>
</tr>
<tr>
<td>$q_i \bar{q}_i \rightarrow q_i \bar{q}_i$</td>
<td>$\leq m_{q_1}$</td>
<td></td>
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</tbody>
</table>
### Putting it to work for discovery

(simulation)

**SM particles at low** $m_{T2}$  

**Squark mass**

---

**MT2 distribution over many events:**

MT2 endpoint structure is weaker than MT (due to more missing information in the event)
MT2 (like MT) is also a mass-space boundary

The MT2(\chi) curve is the boundary of the region of (mother, daughter) mass-space consistent with the observed event!

MT2 and MT behave in exactly the same way as each other, and consequently they share the same kink structure.
A different kind of MT2

Is this good value?

Google says "shat min" also connected to trousers ["pants"]
MT2 and MT behave in exactly the same way as each other, and consequently they share the same kink structure.

Somewhat surprisingly, MT and MT2 kink-based methods are the only(*) methods that have been found which can in principle determine the mass of the invisible particles in short chains! (see arXiv:0810.5576)

(*) There is evidence (Alwall) that Matrix Element methods can do so too, though at the cost of model dependence and very large amounts of CPU.
That last statement should worry you.

$$m_T(\chi)$$

Case 2

arXiv: 0711.4008

Weighing Wimps with Kinks at Colliders arXiv: 0711.4008

Spot the kink
Are kinks observable?

- Expect KINK only from UTM Recoil (perhaps only from ISR!)
- Expect stronger KINK due to both UTM recoil, AND variability in the visible masses.

Digression

(Salutary Tale – how not to generalise to dissimilar parent and daughter masses)
Cricket

The Stumps
The Ashes

Kinematic Boundary

pitch
twicketkeeper
bowling crease
batsman

delivery
popping crease

bowler
umpire
return crease
wicket
"final test" = “Last cricket match in a series of five or more played over a month when countries’ teams compete”

Can England’s batsmen defeat the Aussie spin bowlers?

How firm was the wicket?

Four runs are scored when the ball reaches the boundary (six if it didn’t hit the ground first)

element – where such calculations are computationally tractable. This final test will show whether it is safe to neglect the effects of spin, determine the character of the creases, and get the desired results by using the boundary.

“final test” = “Last cricket match in a series of five or more played over a month when countries’ teams compete”

How firm was the wicket?

Can England’s batsmen defeat the Aussie spin bowlers?

Four runs are scored when the ball reaches the boundary (six if it didn’t hit the ground first)
Moral

- Call the paper what it does
- or choose a sport that more people play
- or try for furry animals?
More hopeful remarks ..... 

“Top Quark Mass Measurement using mT2 in the Dilepton Channel at CDF” (arXiv:0911.2956 and PRD) reports that the mT2 measurement of the top-mass has the “smallest systematic error” in that channel – under study by ATLAS
Top-quark physics is an important testing ground for mT2 methods, both at the LHC and at the Tevatron. If it can’t work there, its not going to work elsewhere.

change of topic!
Not all proposed new-physics chains are short!

Plot distributions of the invariant masses of what you can see

(more details in arXiv:1004.2732)

If chains are longer use “edges” or “Kinematic endpoints”
What is a kinematic endpoint?

• Consider $M_{LL}$

• Zoom in on di-leptons to calculate $m_{LL}$

• In slepton rest-frame

\[ m_{\tilde{L}}^2 = (m_{\tilde{L}}^{max})^2 (1 - \cos \theta)/2 \]
Exercises

• (8) Prove that the phase space distribution for the $M_{LL}$ invariant mass is has the triangular shape shown on the previous slide, and

• (9) Show that the endpoint is located at

$$\left(m_{ll}^{\text{max}}\right)^2 = \left(\frac{m_2^2 - m_{R}^2}{m_R^2}\right)\left(\frac{m_R^2 - m_{L}^2}{m_R^2}\right)$$
Note key difference to bounding vars

• With the bounding vars you **place a bound on** a property/parameter/invariant of the hypothesis or model by construction.

• With the kinematic edges and endpoints, you look for a kinematic structure in a distribution, and use it to **constrain one or more parameters** of the hypothesis or model.

What about these invariant masses?
Some extra difficulties – may not know order particles were emitted

Therefore need to define order-blind variables such as

\[ m_{ql}^{\text{high}} = \max[m_{ql^+}, m_{ql^-}] \]
\[ m_{ql}^{\text{low}} = \min[m_{ql^+}, m_{ql^-}] \]

\[ m_{j\ell(s)}^2(\alpha) \equiv \left( m_{j\ell_n}^2 + m_{j\ell_f}^2 \right)^{\frac{1}{\alpha}} \]
\[ m_{j\ell(d)}^2(\alpha) \equiv \left| m_{j\ell_n}^2 - m_{j\ell_f}^2 \right|^{\frac{1}{\alpha}} \]

Determine how edge positions depend on sparticle masses

<table>
<thead>
<tr>
<th>Related edge</th>
<th>Kineastic endpoint</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t^+t^-$ edge</td>
<td>$(m_{\tilde{t}}^2) = (\bar{t} - \bar{t})(\tilde{t} - \chi)/\bar{t}$</td>
</tr>
<tr>
<td>$t^+\ell q$ edge</td>
<td>$(m_{\tilde{t}}^2) = \max \left[ (m_{\tilde{t}} - m_{\tilde{q}})^2, (m_{\tilde{t}} - m_{\ell})^2, (m_{\tilde{t}} - m_{\chi})^2 \right]$</td>
</tr>
<tr>
<td>$X q$ edge</td>
<td>$(m_{\tilde{t}}^2) = X - (\bar{t} - \chi)(\bar{t} - \chi)/\bar{t}$</td>
</tr>
<tr>
<td>$t^+t^-$ threshold</td>
<td>$(m_{\tilde{t}}^2) = \left{ \begin{array}{ll} 2(\bar{t}(\bar{t} - \chi) - \chi) + (\bar{t} + \chi)(\bar{t} - \chi)(\bar{t} - \chi) \ -2(\bar{t} - \chi)(\bar{t} + \chi)(\bar{t} + \chi) \end{array} \right.$</td>
</tr>
<tr>
<td>$t^+\ell q$ edge</td>
<td>$(m_{\tilde{t}}^2) = (\bar{t} - \chi)(\bar{t} - \chi)/\bar{t}$</td>
</tr>
<tr>
<td>$t^+q$ edge</td>
<td>$(m_{\tilde{t}}^2) = (\bar{t} - \chi)(\bar{t} - \chi)/\bar{t}$</td>
</tr>
<tr>
<td>$t^+q$ low-edge</td>
<td>$(m_{\tilde{t}}^2) = \max \left[ (m_{\tilde{t}} - m_{\ell})^2, (m_{\tilde{t}} - m_{\chi})^2 \right]$</td>
</tr>
<tr>
<td>$M_{T2}$ edge</td>
<td>$\Delta M = m_{\tilde{t}} - m_{\chi}$</td>
</tr>
</tbody>
</table>

Table 4: The absolute kinematic endpoints of invariant mass quantities formed from decay chains of the types mentioned in the text for known particle masses. The following shorthand notation has been used: $\bar{t} = m_{\tilde{t}}, \chi = m_{\chi}, \ell = m_{\tilde{\ell}}, q = m_{\tilde{q}}$ and $X$ is $m_{\tilde{X}}$ or $m_{\chi}$ depending on which particle participates in the "smashout" decay.

So now we have:

<table>
<thead>
<tr>
<th>Large set of measurements</th>
<th>Theoretical expressions for edge positions in terms of masses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Endpoint</td>
<td>S5</td>
</tr>
<tr>
<td>$t^+t^-$ edge</td>
<td>109.10</td>
</tr>
<tr>
<td>$t^+t^-q$ edge</td>
<td>532.1</td>
</tr>
<tr>
<td>$t^+q$ high-edge</td>
<td>483.5</td>
</tr>
<tr>
<td>$t^+q$ low-edge</td>
<td>321.5</td>
</tr>
<tr>
<td>$t^+\ell q$ threshold</td>
<td>266.0</td>
</tr>
<tr>
<td>$X q$ edge</td>
<td>514.1</td>
</tr>
<tr>
<td>$\Delta M$ ($M_{T2}$ edge)</td>
<td></td>
</tr>
</tbody>
</table>
**Fit** all edge position for masses!

...mainly constrain mass differences

Typical scatter of results of fit might look like this in mass space.

Cross section information is orthogonal to mass differences.
How applicable are these long chain techniques?

For the chain $\bar{q} \rightarrow q\tilde{\chi}_2^0 \rightarrow q\tilde{t}_R \rightarrow q\tilde{u}_1^0$
we need:

- $m_{\tilde{X}_2^0} > m_{\tilde{t}_R} > m_{\tilde{X}_1^0}$
- $m_{\tilde{q}} > m_{\tilde{q}}$

This is possible over a wide range of parameter space.

If this chain is not open, the method is still valid, but we need to look at other decay chains.

Example mSUGRA inspired scenario: $-A_0 = m_0$, $\tan \beta = 10$, $\mu > 0$

Other ambiguities

\[(m_{\ell q})_{\text{max}}^2 = \begin{cases} (m_q - m_{\chi_1})^2 & \text{if } m_{\chi_2}^2 > m_q m_{\chi_1}^2 \\ (m_q^2 - m_{\chi_2}^2)(m_{\chi_1}^2 - m_{\chi_2}^2)/m_{\chi_2}^2 & \text{otherwise} \end{cases}\]

Both look the same to the detector

(Though shape differs – see later)

Endpoints are not always linearly independent

e.g. if \( m_{\tilde{q} L} > m_{\chi_2}^2/m_{\chi_1} \) and \( m_{\chi_1}^2 + m_{\chi_2}^2 > 2 m_{\chi_1} m_{\chi_2} > 2 m_{\tilde{q} L} \),

then the endpoints are

\[
\begin{align*}
(m_{\tilde{q} l}^\text{max})^2 &= (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)(m_{\tilde{q}_1}^2 - m_{\tilde{q}_3}^2)/m_{\tilde{q}_2}^2 \\
(m_{\tilde{q} l}^\text{max})^2 &= (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)(m_{\tilde{q}_1}^2 - m_{\tilde{q}_3}^2)/m_{\tilde{q}_1}^2 \\
(m_{\tilde{q} l}^\text{max})^2 &= (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)(m_{\tilde{q}_1}^2 - m_{\tilde{q}_3}^2)/m_{\tilde{q}_2}^2 \\
(m_{\tilde{q} l}^\text{max})^2 &= (m_{\tilde{q}_1}^2 - m_{\tilde{q}_2}^2)(m_{\tilde{q}_1}^2 - m_{\tilde{q}_3}^2)/m_{\tilde{q}_1}^2 \\
\Rightarrow (m_{\tilde{q} l}^\text{max})^2 &= (m_{\tilde{q} l}^\text{max})^2 + (m_{\tilde{q} l}^\text{max})^2
\end{align*}
\]

Four endpoints not always sufficient to find the masses

- Introduce new distribution \( m_{\tilde{q} l}^\text{min} \) identical to \( m_{\tilde{q} l}^\text{max} \) except require \( \theta > \pi/2 \)

It is the minimum of this distribution which is interesting

Slide from David Miller
Different parts of model space behave differently: $m_{\text{QLL}}^{\text{max}}$

Where are the big mass differences?

$$m_{\text{max}}^{\text{max}} = \begin{cases} \max [q(-\xi)(\xi-x), (u-d)(\xi-x), (u-d)^2/\xi] \\ \text{except for the special case in which } p^2 < q^2 < q^2 \text{ and } \xi^2 < q^2 \text{ where one must use } (m_q - m_{\tilde{q}})^2. \end{cases}$$

Exercise

• (10) Prove either

$$m_{\text{max}}^{\text{max}} = \begin{cases} (m^2_{q^2} - m^2_{q^2})(m^2_{\tilde{q}^2} - m^2_{\tilde{q}^2})/m^2_{\tilde{q}^2} & \text{iff } m_{q^2}^2 < m_{q^2}m_{\tilde{q}^2}, \\ (m^2_{q^2} - m^2_{q^2})(m^2_{\tilde{q}^2} - m^2_{\tilde{q}^2})/m^2_{\tilde{q}^2} & \text{iff } m_{q^2}m_{\tilde{q}^2} < m^2_{q^2}, \\ (m^2_{q^2} - m^2_{q^2})(m^2_{\tilde{q}^2} - m^2_{\tilde{q}^2})/(m^2_{\tilde{q}^2}m^2_{\tilde{q}^2}) & \text{iff } m^2_{q^2} < m_{q^2}m^2_{\tilde{q}^2}, \\ (m^2_{q^2} - m^2_{q^2})^2 & \text{otherwise}. \end{cases}$$

or

$$m_{\text{max}}^{\text{max}} = \begin{cases} \max [(u-d)(\xi-x), (u-d)(\xi-x), (u-d)^2/\xi] \\ \text{except for the special case in which } p^2 < q^2 < q^2 \text{ and } \xi^2 < q^2 \text{ where one must use } (m_q - m_{\tilde{q}})^2. \end{cases}$$

and show that they are equivalent.

(See definitions of symbols approx three slides back).
Which parts of $(m^2_{ql\text{near}}, m^2_{ql\text{far}}, m^2_{ll})$-space are populated by these events:

Answer: The Vegetable Samosa
Can see II edge clearly.

Can touch $m_{IIq}$ sphere at carrot corner
Can touch $m_{llq}$ sphere at onion corner

Can touch $m_{llq}$ sphere at noodle corner

Christopher Lester
Can touch $m_{\perp r}$ sphere on the “front”

So, in principle, find masses by looking for highest contrast edge.

Distribution for correct mass hypothesis

Distributions for incorrect mass hypotheses
Exercise

(11) For fixed masses of the four particles on the SUSY backbone, find a function \( f(q^\mu, l_{\text{near}}^\mu, l_{\text{far}}^\mu) \) that is zero on the surface of the samosa, and is non-zero elsewhere.

[Hint: I suggest you try to solve for the invisible LSP momentum as a linear combination of the three visible four-momenta \( q^\mu, l_{\text{near}}^\mu, l_{\text{far}}^\mu \) and a fourth four-vector that is a totally antisymmetric combination of them \( \Omega^\mu = \epsilon^{\mu\nu\rho\sigma} q^\nu l_{\text{near}}^\rho l_{\text{far}}^\sigma \). Then see under what conditions this solution is meaningful.]

The “shadow” (projection) of the samosa is useful for origami too

\[ \text{Figure 7: Obtaining the shape of the } m^2_{j(\text{top})} \text{ versus } m^2_{j(\text{bot})} \text{ bivariate distribution by folding the } m^2_{j(\text{top})} \text{ versus } m^2_{j(\text{bot})} \text{ distribution across the line } m^2_{j(\text{top})} = m^2_{j(\text{bot})}. \] This particular example applies to region \( R_0 \). For the other three regions, refer to Figs. 8(a), 8(b) and 8(c).
Formalising an old idea ... kinematic boundaries, creases, edges, cusps etc

FIG. 1: A schematic diagram describing the relation between the full phase space and the projected observable phase space.
Adding even more assumptions …

Let’s consider what happens when we allow ourselves to look at more than one event ….

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N successive 2-body decays

- In D space-time dimensions

- \(D + (N+1)\) unknowns: comprising
  - D unknown momentum-components for final "missing particle"
  - \((N+1)\) unknown backbone-particle masses

- \(N+1\) constraints:
  - Invariant masses of the backbone-\textit{momenta} must match the "unknown" masses

- \(\text{UNKNOWNS} - \text{CONSTRAINTS} = D > 0\)
  - Cannot solve for unknowns! \(\heartsuit\)

Why not look at K events?

- \(K\) events, each \((N\) successive 2-body decays\)

- \(KD + (N+1)\) unknowns: comprising
  - \(KD\) unknown momentum-components for final "missing particle"
  - \((N+1)\) unknown backbone-particle masses

- \(K(N+1)\) constraints:
  - Invariant masses of the backbone-\textit{momenta} must match the "unknown" masses

- \(\text{UNKNOWNS} - \text{CONSTRAINTS} = K(D - (N + 1)) + (N + 1)\)

- System solvable for \(K \geq \frac{N + 1}{N + 1 - D}\) provided \(N + 1 > D\) i.e. \(N \geq 4\).
Ambiguities

• Which jet is which?
• Which lepton is which?

• So will need more events than the last calculation suggests ~ x4 ?

“Mass relation” method: summary

• Can:
  – reconstruct complete decay kinematics
  – Measure all sparticle masses

• provided that:
  – Chain has \( N \geq 4 \) successive two-body decays
  – One simultaneously examines at least

\[
\frac{N + 1}{N + 1 - D} = \frac{N + 1}{N - 3}
\]

events sharing the same sparticles.
Some example reconstructed masses
(100 events, toy MC)

See sections X and IX of hep-ph/0402295

Caveats:
Nobody has shown that this will work for real data.
Sample purity. Bias. Heavily model dependent?

Dependence on reconstruction resolution.

N=4 two-body decays

• Fewer than 5 events
  – Under constrained, cannot solve

• 5 events
  – Can solve in principle (ignoring ambiguities)
  – Can treat events as “ideal”

• More than 5 events
  – Over constrained. Potential for inconsistency.
  – Reconstructed events will not “make sense” until resolutions are taken into account.

Though see Miller hep-ph/0501033
Nobody has shown that this will work for real data.
Another sort of “just”-constrained event
– get constraint from other “side”

- Even if there are invisible decay products, events can often be fully reconstructed if decay chains are long enough.
- (mass-shell constraints must be >= unknown momenta)
- Since we can use ptmiss constraint, chains can be shorter than N=4 now.

Left: case considered in hep-ph/9812233

Or do both at once
– pairs of double events!

- Pairs of events of the form:

are exactly constrained.
(arXiv:0905.1344)
What about shapes of distributions?

Compare shapes of invariant mass distributions for the highlighted pairs of visible massless momenta:

versus
One piece of information (the endpoint position) is not sufficient to determine $M_A$, $M_B$ and $M_C$. 
Do we have enough information from shape alone to find $M_A$ and $M_B$ in this three body decay, then?

Yes and no ..

- Putting aside experimental fears concerning efficiency and acceptance corrections …
- … huge errors in the fit, and very poor sensitivity to absolute mass scale. See next exercises.
- This is why endpoints, edges and resonances are good, but shapes less so.
Exercises

• (12) Determine the shape of the phase space distribution $d\sigma/d(m_{ll})$ (up to an arbitrary normalizing constant) for the three-body decay shown below. Assume massless visibles, and arbitrary masses for the parent and invisible.

• (13) Prove that $r=x/y$ must lie in the range $1/\sqrt{3} \leq r \leq 1/\sqrt{2}$. (Note this means $r$ can only move by $\pm 0.06$ ... not far!)

• (14) Estimate how many events (approximately) would be needed to distinguish two $r$ values differing by 0.012 (i.e. $\sim 1/10^{th}$ of allowed range)

At fixed $M_A-M_B$ you should find

- $M_B=0$
- $M_B=2$
- $M_B=4$
- $M_B=\infty$
The most detailed “shape” of all is the complete likelihood of the data

- Alwall et.al. (arXiv:0910.2522, arXiv:1010.2263) applied matrix element method to:

  - For ~ 100 events get valley in likelihood surface with same shape as boundary of MT2 distribution

Have only begun to scrape the surface. Need an index.

(more details in arXiv:1004.2732 )
Not time to talk about many things

- Parallel and perpendicular MT2 and MCT
- Subsystem MT2 and MCT methods
- Solution counting methods (eg arXiv:0707.0030)
- Hybrid Variables
- Phase space boundaries (arXiv:0903.4371)
- Cusps and Singularity Variables (Ian-Woo Kim)
- Why wrong solutions are often near right ones (arXiv:1103.3438)
- Razors
- and many more!

I have only scratched the surface of the variables that have been discussed. Even the recent review of mass measurement methods arXiv:1004.2732 makes only a small dent in 70+ pages. However it provides at least an index …

Let’s stop here!
Take home messages

• Lots of approaches to kinematic mass measurement
  – some very general, some very specific.
  – very little of the “detailed stuff” is tested in anger. Experimentalists not universally convinced of utility!
  – very often BGs present serious impediment.
  – theorists and experimenters should pay close attention to zone of applicability

• BUT
  – Finding sensible variables buys more than just mass measurements - e.g. signal sensitivity

Extras if time …
Other MT2 related variables (1/3)

• **MCT** ("Contralinear-Transverse Mass")
  (arXiv:0802.2879)
  – Is equivalent to MT2 in the special case that there is no missing momentum (and that the visible particles are massless).
  – Proposes an interesting multi-stage method for measuring additional masses
  – Can be calculated fast enough to use in ATLAS trigger.

Other MT2 related variables (2/3)

• **MTGEN** ("MT for GENeral number of final state particles")
  (arXiv:0708.1028)
  – Used when
    • each "side" of the event decays to MANY visible particles (and one invisible particle) and
    • it is not possible to determine which decay product is from which side … all possibilities are tried

• **Inclusive or Hemispheric MT2** (Nojirir + Shimizu) (arXiv:0802.2412)
  – Similar to MTGEN but based on an assignment of decay product to sides via hemisphere algorithm.
  – Guaranteed to be >= MTGEN
Other MT2 related variables (3/3)

- **M2C** (“MT2 Constrained”) arXiv:0712.0943 (wait for v3 ... there are some problems with the v1 and v2 drafts)
- **M2CUB** (“MT2 Constrained Upper Bound”) arXiv:0806.3224

- There is a sense in which these two variables are really two sides of the same coin.
  - if we could re-write history we might name them more symmetrically
  - I will call them \( m_{\text{Small}} \) and \( m_{\text{Big}} \) in this talk.

**m\text{Small} and m\text{Big}**

- Basic idea is to combine:
  - MT2
  - with
  - a di-lepton invariant mass endpoint measurement (or similar) providing:
    \[ \Delta = M_A - M_B \]
    (or \( M_Y - M_N \) in the notation of their figure above)
"Best case"  
(needs SPT, i.e. large recoil PT)  
Both $m_{\text{Big}}$ and $m_{\text{Small}}$ are found.

"Typical ZPT case"  
(no $m_{\text{Big}}$ is found)
“Possible ZPT case” (neither $m_{\text{Big}}$ nor $m_{\text{Small}}$ is found)*

* Except for conventional definition of $m_{\text{Small}}$ to be $\Delta$ in this case.

“Possible SPT case” (no $m_{\text{Small}}$ is found)*

* Except for conventional definition of $m_{\text{Small}}$ to be $\Delta$ in this case.
What $m_{\text{Small}}$ and $m_{\text{Big}}$ look like, and how they determine the parent mass

Here is the true value of the parent mass ... determined nicely

Outcome:

• $m_{\text{Big}}$ provides the first potentially-useful event-by-event upper bound for $m_A$
  – (and a corresponding event-by-event upper bound for $m_B$ called $m_{\chi_{UB}}$)

• $m_{\text{Small}}$ provides a new kind of event-by-event lower bound for $m_A$ which incorporates consistency information with the dilepton edge

• $m_{\text{Big}}$ is always reliant on SPT (large recoil of interesting system against “up-stream momentum”) – cannot ignore recoil here!
LHC Specific problems

- Hadron Collider – z-boost of COM unknown
- Pile up, multiple interactions
- Production of many new particles at once?

- Multiple massive stable invisible particles?
What sort of parameter spaces?

- High dimensional
- At the very least, 8 dims
- More like ~100 dims

- No really compelling reasons to believe in any particular simple model

### SUSY params
- $m_0$
- $M_{1/2}$
- $A_0$
- Tan beta
- $\text{Sgn } \mu$

### SM params
- $m_b$
- $m_t$
- $\alpha_s(M_Z)$

Unusual parameter spaces!

Shape of typical set is often something quite horrible.
Contrast with UA1/UA2

  - Predictions in terms of (then) unknown $\theta_W$:
    - $M_Z > 75 \text{ GeV}/c^2$, $M_W > 35 \text{ GeV}/c^2$
- By 1982 $\theta_W$ much constrained, giving:
  - $M_Z \approx 92 \pm 2 \text{ GeV}/c^2$, $M_W \approx 82 \pm 2 \text{ GeV}/c^2$
- CERN able to build UA1+UA2 (~1980) knowing the above.
- In 1983 UA1+UA2 observe $W$ and $Z$ at expected masses:
  - $M_Z \approx 95 \pm 3 \text{ GeV}/c^2$, $M_W \approx 81 \pm 5 \text{ GeV}/c^2$

I.W.Kim: “Algebraic singularity method of mass measurement with missing energy”

Formalising an old idea ... kinematic boundaries, creases, edges, cusps etc

FIG. 1: A schematic diagram describing the relation between the full phase space and the projected observable phase space.
But outside standard model

• Don’t usually know mass of invisible final state particle!
  • (neutralino?)

So for new physics need:

• Chi parameter “χ” to represent the hypothesized mass of invisible particle

Chi parameter “χ” (mass of “invisible” final state particle) is EVERYWHERE!

(most commonly on x-axis of many 2D plots which occur later)
Reminder:

We define the “full” transverse mass in terms of “$\chi$”, a hypothesis for the mass of the invisible particle, since it is unknown.

$$m_T^2(\chi) = m_{vis}^2 + \chi^2 + 2(E_{Tvis}E_{Tmiss} - p_{Tvis}p_{Tmiss})$$

where

$$E_{Tvis} = m_{vis}^2 + p_{Tvis}^2$$

and

$$E_{Tmiss}^2 = \chi^2 + p_{Tmiss}^2$$