"Particles Beyond The Standard Model"

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SM Particle Content:

They can be classified according to their quantum numbers.

Colorless

\[
\begin{array}{c|c|c|c|c}
\nu_e & \nu_\mu & \nu_\tau & \gamma & H \\
\hline
\ell & \mu & \tau & Z & W \\
\end{array}
\]

Carry

\[
\begin{array}{c|c|c|c|c|c}
\textit{u} & \textit{c} & \textit{t} & \textit{g} \\
\hline
\textit{d} & \textit{s} & \textit{b} \\
\end{array}
\]

fermions, spin-$\frac{1}{2}$, bosons, boson, Spin-1 Spin-0

The electron was discovered first in 1897 by J.J. Thomson.

The Higgs was discovered most recently in 2012.
(In fact, this is the first TABI after the discovery of the Higgs.)

The particle that was discovered the “2nd earliest” was muon, by Carl Anderson in 1936.

At the time, I. I. Rabi famously asked “Who ordered that?” for it was not clear what it was good for.

Nowadays we know there are three generation of elementary particles whose quantum numbers are identical. The only difference is their masses.

We still don’t know why there are three generations.
So, really, why do we need particles beyond the SM when we don’t even know what muon is good for ??

There are in fact two reasons to expect particles beyond the SM:

1) Empirical Evidence —

Dark matter — no candidate for CDM
Baryon Asymmetry — CKM picture does not provide enough CP violation.
Neutrino Oscillations — may need RH neutrinos?

2) Theoretical Prejudice —

The celebrated naturalness problem:

The Higgs mass is quadratically sensitive to UV physics unless new particles come in at ~ 1 TeV to soften the sensitivity.
In these set of lectures, I would like to emphasize the naturalness point of view.

It is a theoretical prejudice, but nonetheless a very well-founded prejudice. Understanding whether TeV scale physics is natural or not would have profound implications on the way we understand the universe.

The naturalness problem in the Higgs mass is often stated in terms of 1-loop corrections to the scalar mass

\[ m_h = (m_{h0}^2) - \frac{c}{16\pi^2} \Lambda^2 \]

\( m_{h0} \): bare parameter, \( \Lambda \): cut off
Since we now know $m_h \approx 125$ GeV, and if the next cutoff is all the way up at $M_{planck} = 10^{19}$ GeV, we have

$$(125 \text{ GeV})^2 = (m_h^0)^2 = \frac{1}{16\pi^2} (10^{19} \text{ GeV})^2$$

So we need a cancellation up to 17 digits to arrive at 125 GeV!  

On the other hand, if the next cutoff is at around 1 TeV:

$$(125 \text{ GeV})^2 = (m_h^0)^2 = \frac{1}{16\pi^2} (1000 \text{ GeV})^2$$

Then we see $O(100 \text{ GeV})$ scale mass is very "natural" and does not require much fine-tuning.
The huge separation of scales between Higgs mass, \( O(100 \text{ GeV}) \), and the next known physical scale, \( M_{\text{Planck}} \approx 10^{19} \text{ GeV} \), is the heart of the (in)famous "Hierarchy Problem".

There are two folklore surrounding the hierarchy problem:

1) Why is this big separation of \( M_h \) and \( M_{\text{Planck}} \) "unnatural"? After all, we have seen other examples of diverse mass scales in the SM.

\[
\text{Eg.: } \frac{M_e}{m_t} \approx \frac{0.5 \times 10^{-3} \text{ GeV}}{175 \text{ GeV}} \sim 10^{-6}
\]
We will see that both folklores are due to a lack of understanding of the (Wilsonian) Renormalization Group.

Let's consider the simple example of a free scalar first (cf. hep-th/9310046, Polchinski '92 TASI)

\[ S_{\text{free}} = \frac{1}{2} \int d^4x \, \partial \phi \partial \phi. \]

Since the action is dimensionless, we see the dimension of \( \phi \) to be \( [\phi] = E^{-1+d/2} \).

Now add to the free lagrangian a set of "local interactions" \( \Theta_i \)

\[ S = S_{\text{free}} + \int d^d x \, \sum_i g_i \Theta_i \to \text{couplings} \]

If \( [\Theta_i] = E^{d/2} \), \( [g_i] = E^{d-\delta_i} \).
2). If we use dimensional regularization instead of a cut-off to regularize the divergent integral, then it is well-known the Dim Reg only see "log-divergence", but not "power divergence".

E.g.,

\[ \int \frac{d^d k}{(2\pi)^d} \frac{1}{k^2 - m^2} \sim \lambda^2. \]

\[ \int \frac{d^d k}{(2\pi)^d} \frac{1}{(k^2 - m^2)} \sim -\frac{1}{d} \left( \frac{d}{2\pi} \right)^d \Gamma(d/2) \left( \frac{1}{m^2} \right)^{1-d/d} \]

\[ \sim m^2 \left( \frac{1}{\lambda} - \log m^2 \right) \]

Therefore Dim Reg is "special" and we would not have the hierarchy problem if we just stick with Dim Reg!
Now define the dimensionless couplings

\[ \lambda_i = \frac{\Lambda_i}{\Lambda}, \quad \Lambda = \text{characteristic scale of the system} \]

\[ \lambda_i \sim \Theta(1) \]

So if we consider a scattering process at the energy scale \( E_0 \), dimensionality analysis suggests the contribution from a given operator \( O_i \) is

\[ g_i \int d^d x \, \mathcal{O}_i \sim g_i (E_0)^{d_i - d} \]

\[ \sim \lambda_i \left( \frac{E_0}{\Lambda} \right)^{d_i - d}. \]

Thus we see if

<table>
<thead>
<tr>
<th>( \delta_i )</th>
<th>Size at ( E \to 0 )</th>
<th>Jargon</th>
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<tbody>
<tr>
<td>(&lt; d)</td>
<td>grows</td>
<td>relevant</td>
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<td>( = d)</td>
<td>constant</td>
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<tr>
<td>( &gt; d)</td>
<td>decreases</td>
<td>irrelevant</td>
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For a scalar mass term

\[ \mathcal{O}_{\phi^2} = \phi^2, \quad \Delta m^2 = 2 \]

\[ g_{\phi^2} \int dx \phi^2 \sim \lambda_{\phi^2} \frac{\Lambda^2}{E^2} \quad \text{in} \; d=4 \]

If \( \Lambda \sim M_{\text{Planck}} \sim 10^{19} \text{GeV} \),

at energy \( E \sim 100 \text{ GeV} \), the natural size of the mass term is then

\[ \lambda_{\phi^2} \times \frac{(10^{19})^2}{(100)^2} \times (100 \text{ GeV})^2 \]

That is, to arrive at a "light mass" for the Higgs boson, we need to start with an extremely tiny

\[ \lambda_{\phi^2} \sim (10^{-17})^2 \]

!!
So for this is just dimensional analysis.

The precise prediction of how a coupling constant evolves (or "runs") with energy is given by the renormalization group equation

\[ \frac{d}{d \log p} \lambda_{\phi^2} = \left[ -2 + \gamma_{\phi^2}(g_i) \right] \lambda_{\phi^2}. \]

\( \lambda \) may depend on all couplings, which "run" and in turn depend on \( \log p \).

In the vicinity of a fixed point where the couplings \( \{g_i\} \) become constant \( \{g_i^*\} \) and do not run,

\[ \lambda_{\phi^2}(p^2) = \lambda_{\phi^2}(\Lambda^2) \left( \frac{\Lambda}{p} \right)^{2 - \gamma_{\phi^2}(g_i^*)}. \]
Under the assumption that $V_0 (g_i^*) \ll 1$, we see the "mass coupling" is related to each other at different energy scales quadratically!

Now one can formulate the hierarchy problem in terms of criticality:

**Phase diagram of EWSB**

![Diagram showing criticality](chart)

$\langle H \rangle \sim M_{Pl}$

$m^2 \geq 0$

$m^2 > 0$

$\langle H \rangle = 0$

We are sitting extremely close to criticality.

Cf. hep-ph/0606105
Gradice + Rattazzi

Why??
Solutions (Answers):

1) The critical line is a locus of enhanced symmetry, which is why we like to sit practically on top of the critical line.

In fact, there are only two known examples of such enhanced symmetries:

i) Bosonic symmetry: (spontaneously) broken global symmetry.

   → The Higgs is a PNRB

ii) Fermionic symmetry: Supersymmetry.

The Higgs could also be the 5th component of a gauge field in extra dimensions. But by AdS/CFT this scenario is dual to PNRB Higgs.
2) \( \Phi^2 \sim O(1) \), so that the mass parameter runs "very slowly". This in turn implies there should be an "approximate" conformal invariance.

In this scenario, since the running \( \beta \) is slow, we do not need to sit close to the critical line to get to 125 GeV.

c.f. Luty & Okui hep-ph/0409274
Rattazzi et al. 0807.0004.

3) Accept the fine-tuning!!
You may wonder: Has naturalness principle ever "predicted" new particles before?

I can give two examples where Naturalness successfully "predicted" existence of new particles.

\(*\): An electron, classically, has a Coulomb potential energy

\[ V(r) = \frac{e^2}{r} \]

Assuming the electron is a point-like particle, the total energy stored in the "self-energy" is infinite

\[ E = \sum \frac{e^2}{r} \rightarrow \infty \]

Since special relativity dictates that energy = mass.
We see that, classically, a point-like particle like the electron should "naturally" be infinitely heavy!

You may object that we don't really know if the electron is a point particle. The best we can do is to talk about the experimentally limit on the size of an electron: \[ r_e < 10^{-18} \text{ m} \]

So even if we cut off the integral at this size, we would get

\[ m_e \sim \frac{e}{r_e} \sim \frac{e}{10^{-18} \text{ m}} \sim 10 \text{ GeV} \]

which is still much larger than the measured \[ m_e \sim 0.5 \text{ MeV} \sim 5 \times 10^{-4} \text{ GeV} \].
The resolution is to introduce "new particles" which have the same mass as the electron but opposite charge — the positron.

Then (V. Weisskopf)

[*] at distance scale of Bohr radius, 10^{-5} m, the Coulomb potential energy is large enough to "pair" create e^+e^- out of vacuum.

[*] Such interactions cancel the infinite self-energy of the electron.

[*] To guarantee the cancellation for EVERY electron in the Universe, we need a new symmetry called "Chiral symmetry."
In the end, Naturalness principle predicts new degrees of freedom and new symmetry principle to cancel the linear divergence in the electron mass.

Indeed, in QFT the 1-loop correction to the fermion mass is only log-divergent

\[ S_m \sim m_e \log \frac{\Lambda}{m_e} \]

More importantly, the correction is controlled by a "symmetry breaking" parameter \( m_e \).

If we set \( m_e \to 0 \), chiral symmetry is restored and \( S_m = 0 \).

Again, the "critical line" is a locus of enhanced symmetry!!
Became the dependence on the cut off is only logarithmic, it one could have a small fermion mass even with a large cutoff, without introducing much fine-tuning.

This dispels the 1st folklore!

The first example is the discovery of pions in low-energy QCD.

\[ \text{Pion is a spin-0 (pseudo) scalar meson} \]

\[ M_{\pi} \sim 150 \text{ GeV} \]

Again from Naturalness, in order for the pion mass to be "natural", new degrees of freedom must exist at or below the scale \[ M_{\text{GUT}} \sim 1 \text{ TeV} \]
We now know

Surely enough, the rho-meson has a
mass \( M \rho \sim 750 \text{ MeV} \) and 1 GeV

is the scale where QCD becomes strongly
coupled and the chiral symmetry in the
light quark sector gets "spontaneously
broken": \( \Lambda_{\text{QCD}} \sim 1 \text{ GeV} \).

We now realize that pion is "much
lighter" than \( M \rho \) and \( \Lambda_{\text{QCD}} \)
because there is a symmetry that is
"non-linearly realized" by 0 pions.

\( \exists \) They are PNGB!
The Higgs boson in the SM face a completely analogous problem:

Interactions of the Higgs with W/Z bosons, top quarks, and itself creates a self-energy that is quadratically sensitive to UV scale:

\[ \Lambda \sim 1 \text{ TeV} \]

\[ 8m_h^2 \sim \frac{9}{64\pi^2} g^2 \Lambda^2 \sim (700 \text{ GeV})^2 \]

\[ 8m_h^2 \sim \frac{1}{16\pi} \lambda \Lambda^2 \sim (500 \text{ GeV})^2 \]

\[ -\frac{3}{8\pi^2} y_t^2 \Lambda^2 \sim -\left( \frac{200 \text{ GeV}}{\Lambda} \right)^2 \]

Top Yukawa
So if there exists new particles and new symmetry principle at 1 TeV, the Higgs could "naturally" be at 125 GeV.

Now we see that naturalness in the Higgs mass requires "new particle beyond the SM."

The question then is How do we search for them?

There are two complementary approaches.

1) Indirect search

2) Direct search.

We will see that they involve different set of assumptions and we need both of them!
Let me first talk about indirect searches.

The starting point is there are some new particles cancelling the quadratic UV-sensitivity in the Higgs mass from SM particles:

\[
\begin{align*}
\text{Oblique corrections:} & \\
\text{PEW, } g_{\nu g} & \\
\text{m}_{\nu} = 173^{+33}_{-13} & \\
\text{m}_{\nu} = 140^{+180}_{-20} & \\
\end{align*}
\]

\[
\phi \times \frac{1}{16\pi^2} \lambda^2 + \frac{M_{\text{new}}}{16\pi^2} + \ldots
\]
It is instructive to ask how one can "observe" the quadratic UV-sensitivity from the $SU(5)$ $W$ boson and top quark $t$?

The trick is to manipulate the diagrams by:

1) Putting one of the external Higgs into its VEV.

2) Attach two photons to the $W$-loop and two photons/gluons to the $t$-top loop.

Now we have $W$/top contributions to $h \rightarrow W$ decays and top contribution to $h \rightarrow gg$ decays!!
There's a very powerful lesson we just learned from this exercise:

If the particles (SM or new alike) contribute to self-energy of the Higgs, carry SM quantum numbers, they will also contribute to $h\rightarrow vv$ and/or $h\rightarrow gj$ decays!

(The reverse statement holds up obviously.)

New particles, if they carry SM quantum number...
We will see this connection is already very useful in constraining new particles. One prime example is the 4th generation quarks.

But first let's see how to compute effects of new particles efficiently by employing the low-energy Higgs theorem, which applies when

\[
\frac{M_{h}^{2}}{4m_{\text{new}}^{2}} < 1.
\]

The statement starts with the assumption that the new particle has "some sort" of coupling to the Higgs:

\[
\begin{array}{c}
\text{Higgs} \quad \text{new} \quad \text{Higgs} \quad \text{new}
\end{array}
\]
The existence of such couplings implies a contribution to the particle mass from Higgs VEV:

\[ m_c = m_0 + \frac{1}{2} \, \text{Higgs} \cdot \text{Scale} \]

for proof. The Higgs field \( \phi \) comes from the operator \( \phi H^{\dagger} H / 8 \).

For vector-like fermions \( \text{Gmass} = C \cdot u \)

For "4th generation" fermion:

In any case, we see that in general

\[ m_{\text{new}} = m_{\text{new}}(u) \]

Then there is a relation between arbitrary matrix elements with and without an external Higgs in the zero momentum limit. The "u" dependence comes through the mass:

\[ \lim_{h \to 0} M(X+h) = \frac{2}{\delta v} M(X) \]
The way the theorem comes about can be seen diagrammatically.

Consider a 2-pt function of scalars

\[ S \equiv M(\cdots) = \frac{1}{p^2 - m_s^2(w)} \]

Then

\[ \frac{2}{2u} \left( M(\cdots) \right) = \frac{1}{p^2 - m_s^2(w)} \cdot \frac{2 m_s^2(w)}{2 u} \cdot \frac{1}{p^2 - m_s^2(w)} \]

\[ = S \]

\[ \frac{2}{2u} \left( m_s^2 + \frac{1}{2} s_u^2 u^2 \right) = \text{Gauge invariant} \]

Exercise:

1) Write down the lowest dimensional operator that would couple the Higgs to a vector-like fermion from which write down \( G_{IHF} \).

2) Repeat 1) for 4th generation fermions.

3) Verify the low energy theorem using 2-pt function of fermions, for both vector-like and 4th generation cases.
Now in the case of Higgs decays.

\[ X + h = \mathcal{O}(\ln m) \]

\[ \Rightarrow X = \mathcal{O}(\ln m) \text{ or } g \mathcal{O}(\ln m) \]

That is, \( X \) is nothing but the self-energy of photons and gluons!

The good news is these two types of two-particle functions are exactly the diagrams we would need to compute for renormalization of electric charge and strong coupling constants, and they are given by the QED and QCD beta functions.

In the end we can compute any new particles contribute to diphoton and dielectron decays almost without any work, as long as you know how to look up their contributions to beta functions.
Let's apply the low-energy theorem to the top quark contribution to $h\to gg$.

The diagram for top contribution to gluon 2-pt function is

$$p^0 \circ \sum_{i=1}^n p^0_i = -\frac{1}{4} \left( \frac{4 \mu^2}{m_T^2} \right)^2 \frac{\alpha_s}{6\pi} \left[ 4 \pi \mu^2 \right]^3 \Gamma(H^3) \frac{1}{2}$$

We have $m_T = \frac{M_T}{\sqrt{2}} \nu$.

Exercise: Verify this in dimensional regularization!

We have

$$h = \frac{\alpha_s}{12\pi} \frac{1}{\nu} \left( G_{\mu}^{\nu} \right)^2$$

$$= \frac{\alpha_s}{16\pi} \left( \frac{4}{3} \right) \frac{1}{\nu} \left( G_{\mu}^{\nu} \right)^2$$

Notice that, strictly speaking, this result is valid only in the limit $M_T^2 \to 0$.

However, in reality, the approximation works surprisingly well.
The width from the approximation is

$$\Gamma(h \rightarrow gg) = \frac{\alpha_s m_h^3}{(28\pi^3)} \left| \frac{1}{6} \cdot \frac{4}{3} \right|^2$$

The exact result, keeping the full $m_t$-dependence:

$$\Gamma(h \rightarrow gg) = \frac{\alpha_s m_h^3}{(28\pi^3)} \left| \frac{1}{6} \cdot A_{1/2}(\frac{4m_t^2}{m_h^2}) \right|$$

$$A_{1/2}(\pi) = 2\pi^2 \left[ \frac{1}{\pi} + (\frac{1}{\pi} - 1) f(\frac{1}{\pi}) \right]$$

$$f(y) = \begin{cases} \arcsin^2 \sqrt{y} & y \leq 1 \\ \frac{\pi}{4} \left( \log \frac{1 + \sqrt{1 - y}}{1 - \sqrt{1 - y}} - i\pi \right)^2 & y > 1 \end{cases}$$

For $m_t = 172 \text{ GeV}$, $m_h = 125 \text{ GeV}$

$$A_{1/2}(\frac{4m_t^2}{m_h^2}) = 1.377 \quad \text{v.s.} \quad \frac{4}{3} = 1.333$$

The difference in the width is $\Delta = 5\%$. 
One feature that is worth mentioning is the "non-decoupling" behavior:

\[ \text{as } M_6 \to \infty, \quad \Gamma(h \to \gamma \gamma) \to 0 \quad ! \]

Another way to see it is the \( \text{d}N_5 \) operator

\[ \frac{dN_5}{d \mathbf{v}} \to \frac{dN_5}{d \mathbf{v}} = \frac{d^5 \mathbf{v}}{c^2 \mathbf{v}} (G_{\text{h}} \mathbf{v})^2 \]

has no \( m_t \)-dependence. It's suppressed by \( \frac{1}{\mathbf{v}} \), not \( \frac{1}{m_t} \) as one would naively expect.

This is because the top quark receives "all" of its mass from the Higgs boson

\[ m_t = \frac{y_t}{\sqrt{2}} \mathbf{v} \]

\[ G_{\text{h}} = \frac{1}{\mathbf{v}} \mathbf{v} \text{ is a SU(2) scalar!} \]

If we hold "\( \mathbf{v} \)" fixed, increasing \( m_t \) means increasing \( y_t \), which in turn increases the Higgs coupling to the top quark. Their effects cancel each other completely!
A better way to understand is to re-write

\[ Q_8 = \frac{\alpha_s}{12\pi} \frac{y_t^2}{\sqrt{2}} \frac{1}{\frac{y_t}{\sqrt{2} v}} h(Q_u^2)^2 \rightarrow \text{Higgs coupling to top}\]

\[ = \frac{\alpha_s}{12\pi} \sqrt{\frac{y_t e}{m_t}} h(Q_u^2)^2 \text{ gluon} \]

This is actually the general formula for colored fermions in the fundamental representation of $SU(3)_c$.

Armed with this, we are ready to draw the first example of particles beyond the SM:

**4th Generations Fermions**

\[ Q_L = \begin{pmatrix} T \\ B \end{pmatrix}_L \]

\[ T, B \]

\[ L = y_t Q L H T \quad + \quad y_B Q B H \]

\[ M_T = \frac{y_t}{\sqrt{2}} u \quad \text{if} \quad m_T, m_B > m_t \]

\[ M_B = \frac{y_B}{\sqrt{2}} u \quad \text{we need} \quad y_T, y_B > y_t = 1. \]
4th generation fermions are certainly very well motivated. (We have 3 generations already. Who says one can't have a 4th one?)

However, in this case, we will see that the Higgs coupling to $gg$ will be significantly enhanced:

$$\mathcal{O}_g^4 = \frac{\alpha_5}{12\pi} \left( \frac{y_t}{m_t} + \frac{y_t}{m_T} + \frac{y_\phi}{m_\phi} \right) \cdot h \left( \frac{\sin^2 \theta}{\Lambda^2} \right)^2$$

$$= 3 \times \frac{\alpha_5}{12\pi} \frac{1}{v} h \left( \frac{\sin^2 \theta}{\Lambda^2} \right)^2.$$

In terms of the width:

$$\Gamma(h \to gg) = 9 \times \Gamma_{SM}(h \to gg).$$

Since the same coupling gives the gluon fusion production rate, the Higgs production cross-section is increased by almost a factor of 10:

$$\sigma(gg \to h) \sim 9 \times \sigma_{SM}(gg \to h).$$

This is very strongly constrained by present measurements at the LHC!!
So it is very difficult to accommodate 4th generation quarks.

What about 4th generation lepton? It won't show up in the gluon fusion production. But it will show up in the diphoton decay of the Higgs. In $h \to \gamma \gamma$, there are two dominant SM contributions:

\[ A_1 \quad A_2 \]

at the amplitude level:

\[ A_1 \left( \frac{q m^2}{m_h^2} \right) : \frac{A_2}{\text{sign}} \left( \frac{q m^2}{m_h^2} \right) = -8.32 : 1.84 \]

Cannot use low-energy theorem.

Thus a 4th generation lepton will decrease the diphoton width by a significant amount.

There is also a theoretical prejudice against 4th generation from naturalness point of view.

This is because 4th generation quark necessarily worsen the fine-tuning in the Higgs mass.
It is very easy to see this

\[ \alpha \left( -\frac{1}{4\pi} \right)^2 \]

Moreover, it is not an accident that 4th generation quarks lead to an enhancement in big coupling, because the interference pattern is correlated:

At this point, one may ask, since 4th generation always worsen the fine-tuning, how can one cook up a model in which the new fermi will "cancel" the top quadratic divergences?

The argument above shows one cannot achieve this cancellation relying on 3-pt vertex of hh\ T\ T\.

Instead, one need to introduce 4-pt coupling
You should feel a bit uneasy here because we are using a dim-5 operator to cancel effects from a dim-4 operator. But this is OK if we can generate the operator in such a way that

\[ m^2 \sim f, \quad \text{i.e.} \quad \frac{m_t}{f} \sim \mathcal{O}(1) \]

Then we don't need unnaturally large \( \Delta t \).

\( \Delta t \) must be related to \( \lambda t \), the top Yukawa coupling.

In EFT, coefficients of different operators are in general not related to each other unless there's a "symmetry" reason.

That is, there must be a symmetry that relates the above two diagrams. This symmetry would relate somehow to the heavier "top partner."
If \( Q \) is the charge corresponding to the
symmetry  \( Q [t+J] \sim T \).

So \( Q \) must be "bosonic" since it relates a
fermion to another fermion.

This is exactly what a "global symmetry" does
if both \( t \) and \( T \) sit in the same
representation of \( G \).

This is the idea of PNVB Higgs: the Higgs
is like the pion, acts as a PNVB under
some spontaneously broken global symmetry.
Both \( t \) and \( T \) transform into each other
under the global symmetry.

Using our general formula we can compute the
effect of "top partner" or higgs coupling:

\[
\sigma = \frac{\lambda v^2}{f^2}
\]

\[
\sigma_{\text{higgs}} \sim \frac{\alpha_s}{12\pi} \left( \frac{1}{m_f^2} \right) \frac{1}{f} \frac{h(f_{\text{true}})^2}{f} \sim m_f^2
\]

Note that their effect decouple like \( m_f^2 \), unlike
4th generation quarks.

Moreover, the relative sign must be "-" if the
Higgs quadratic divergence were to cancel.
Now we can play the same game if it were a new scalar responsible for cancelling the quadratic divergence.

There are some subtle differences though. For example, the relative "sign" the Higgs 2-pt is "flipped" because the fermion loop has an extra "-" sign because of spin-statistics.

\[ t \quad + \quad \text{for cancellation} \]

But now in the case of H-Higgs, the scalar loop has two diagrams:

\[ h \quad \text{for} \quad \text{cancellation} \]

\[ h \quad \text{for} \quad \text{cancellation} \]
So the interference pattern now becomes "constructive interference" if the quadrature divergences are cancelled.

Therefore in SUSY, top squarks interfere constructively or set top in $h \to gg$, when there is no mixing between the stops.

Turning on large mixing will reverse the pattern of interference from being constructive to destructive.

Now we see that naturalness motivates existence of "partners" of top quark $T$ or $T'$. It is then important to search for them in colliders. Moreover, naturalness again suggests that their masses cannot be
heavier than 1 TeV, otherwise the Higgs mass is fine-tuned. So the chance of them being discovered at the LHC seems promising, if they exist.

For collider searches, it is also important to know what they decay to. Given that they are "partners" of the top, it's also natural that they decay to 3rd generation quarks like t and b.

Current search limits for me:

\[ m_t > 600 - 650 \text{ GeV} \]

\[ m_b > 700 - 800 \text{ GeV} \]

So the LHC has great hopes in the next 5 years to "close the gap" on
Discuss whether (fermionic or scalar) top partners.

This is why sometimes you hear people saying that we are at the horn facing a fork in the road" in terms of understanding whether TeV scale physics is "natural" or not.

So far I only discussed cancelling the quadratic divergence in the top sector.

Obviously in any "complete" model, one would also need the cabal quadratic divergences in the W-loop and Higgs quartic loop.
For example, let's consider the $W'$-loop.

$W'/Z$  
\[ h \quad \text{ and } \quad h \]

$W'$ and $Z'$ are heavier cousins of SM $W$, $Z$ bosons.

Moreover, the couplings must come in opposite sign.

However, it's well-known that massive spin-1 bosons require UV completion at

the scale \( \frac{4\pi M_{W'}}{g} \).

Notice that I didn't say it "breaks" gauge invariance, because we can just assume there's no gauge invariance associated with $W'$ and $Z'$.
The way to see this is to look at the high energy limit of the longitudinal scattering of massive gauge bosons:

\[ W^+ W^- \rightarrow W^+ W^- \]

Since the polarisation vector of the longitudinal component is

\[ \varepsilon^L \sim \frac{k_n}{m_W} \]

namely the amplitude scales like

\[ g^2 \left( \frac{E}{m_W} \right)^4 \]

This amplitude becomes strong at

\[ g^2 \left( \frac{E}{m_W} \right)^4 \sim 4\pi \]

or

\[ E \sim \left( \frac{4\pi}{g} \right)^{1/8} m_W \]
However if we insist on a kinetic term that is transverse \( T(F_{\mu \nu}) \), then there are cancellations between the 3-pt and 4-pt vertices, in the end the growth is only

\[
g^2 \left( \frac{E}{m_W} \right)^2 \sim 4 \pi
\]

\[
\Rightarrow \quad E \sim \sqrt{4\pi} \frac{m_W}{g}
\]

which is not very far from above \( m_W \)!

This violation of unitarity is already present in the SM w/o the Higgs boson. The introduction of the Higgs "cancels" the bad \( \frac{E^2}{m_W} \) behavior further, which why people often said the Higgs unitarizes \( WW \) scattering.
The easiest way to UV-complete W' Z' is to introduce extra copies of SU(2) gauge group that is spontaneously broken, just like the SM W/Z bosons.

However, these extra SU(2) group cannot be a "Stand alone" SU(2) group, for otherwise W' Z' would not be charged under electroweak $SU(2)_L \times U(1)_Y$.

Therein the possibility to implement this is to utilize $SU(2)_L \times SU(2)_Z \rightarrow SU(2)_V$.

And take $SU(2)_V = SU(2)_L$ in the SM.

This also ensures that there is a gauge anomaly, it turns out that we need the Higgs to be charged under both gauge group.
It takes some clever model-building to come up with a mechanism to cancel the cancellation of quadratic divergences. This class of model is exactly the so-called "Gut-like Higgs" models, which Seckel talked about.

On the other hand, it is possible to cancel the $W$-loop contribution using fermions:

It's natural to use fermions in the adjoint representation of $SU(2)_L$, thus the standard model dictates that the other fermion in the loop must be in the fundamental of $SU(2)_L$, carrying the same hypercharge as the Higgs.
Thus, we can think of $\tilde{W}$ as the partner of $W$ boson → the "wind" and $\tilde{H}$ as the partner of the Higgs → the Higgsino!

This concept is naturally incorporated in SUSY.

So now we have the following list of new particles to search for:

\[
\{ \tilde{W}, \tilde{Z}, \tilde{h}, \tilde{\tau}^0, \tilde{\tau}^\pm \}
\]

These particles do not carry $SU(3)$ color.

So in general their production rate is much smaller than those "colored" particles simply.

However, in some cases, if they couple to SM quarks and leptons, the bound could be very strong.
e.g., if the $Z'$ couples to SM quarks and leptons, it could be produced through the Drell-Yan process and then decay to $e'^- e^- e^+ e^-$.

This would show up as a spectacular "bump" in $M_{Z'}$. Current bound

$$M_{Z'} \geq 2-4 \text{ TeV}!$$

However, it's quite simple to evade this bound if we simply postulate a new "parity" at the TeV scale, i.e., all new particles are odd under the parity and SM are even.
Examples of a new parity:

\[ \text{SUSY} \rightarrow R\text{-parity} \]
\[ \text{Extra-dim} \rightarrow \text{KK}\text{-parity} \]
\[ \text{little Higgs} \rightarrow T\text{-parity} \]

Then the bound on new particles in general are very weak. This is the case.

So in the end we see there's still room for new particles to survive current search limits, especially for non-colored particles.

To sum up, if we discover any new particles, we should be asking the same old question: Rabi asked, "Who ordered them?"

One of the most important questions would be: "Did Naturalness order the new particles?"